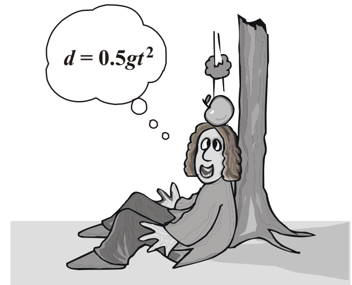
Quadratic Applications

Section 16.3
Quadratic Word Problems:
Motion Applications



The very mention of the word physics is enough to strike fear in the heart of the average high school student. It's not because the concepts are all that hard — it's because the math is so *scugly*! That's scary and ugly all rolled into one. The good news is that many of the formulas used to solve physics applications are quadratics. And you know how to solve quadratics, right? Well, by now you should.

This won't be a physics lesson, but there are a few *scugly* application problems that you should see. Have courage! You can do the math. And you won't even have to come up with the equation; we'll give you that. All you have to do is decide how to solve it and find the right answer.

According to some, Sir Isaac Newton, noted mathematician, was sitting under an apple tree when an apple fell. Because of Newton's contributions to the field of science, you now know that the force that caused the apple to fall is called gravity. The speed of the apple is caused by the acceleration due to gravity.

Example 1: **Let's say Newton saw an apple begin to fall from the top of a tree 11 meters above his head. How long did he have to move out of the way before the apple would have hit him in the head? Use the formula below.**

$$d = 0.5gt^2$$

d = 11 meters (distance the apple fell)
g = 9.8 m/s^2 (acceleration due to gravity)
t = time in seconds for the apple to fall

Step 1: Substitute the values you know into the formula *including the units*.

$$d = 0.5gt^2$$
$$11 \text{ m} = 0.5\,(9.8 \text{ m/s}^2)\,t^2$$

Step 2: Do the math. Multiply the 0.5 and the 9.8 m/s^2.

$$11 \text{ m} = (4.9 \text{ m/s}^2)\,t^2$$

Step 3: Divide by sides by 4.9 m/s^2. The "m/s^2" is a rate, so treat the units like a fraction. Remember that when you divide by a fraction, you reverse the numerator and denominator. The meters cancel. Round to the nearest hundredth.

$$\frac{11\,\cancel{m}}{4.9} \bullet \frac{s^2}{\cancel{m}} = t^2$$

$$2.24 \text{ s}^2 = t^2$$

Step 4: Now you can take the square root of both sides to solve for t. Notice that when you take the square root of s^2, you get just seconds. It takes the apple about 1.5 seconds to fall 11 meters, so Newton has only 1.5 seconds to get out of the way!

$$1.5 \text{ s} = t$$

Example 1 is a fairly easy one. The motion is in only one direction. But if you start throwing things up in the air, they come back down at the same rate as the apple. Now, you have two directions: first up with the speed you threw it and then down with the speed due to gravity. When something goes up and then comes back down, its movement is called projectile motion.

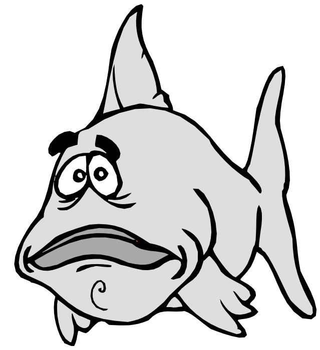
Let's say you threw a rock straight up into the air at 8 m/s from a height of 1.5 meter. How long, in seconds, will it take before the rock hits the ground, or $h = 0$? Without getting too technical, the 8 and the 1.5 go into the projectile motion equation as shown on the right. If you set $h = 0$, that's all you need to know to solve a problem like this one.

$$h = -4.9t^2 + 8t + 1.5$$

$$0 = -4.9t^2 + 8t + 1.5$$

Non-linear Functions

Section 26.2
Quadratic Equations From Graph and Tables



Now that you know how to find the information about quadratic functions from a graph or a table, let's see what you can do with it.

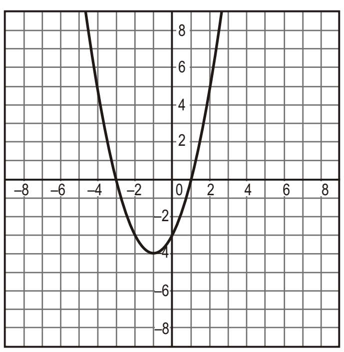
Let's say you have a graph or a table and need to match it to its equation. To match a graph or table to a quadratic equation, you need to check at least three points. You could take each set of integer coordinates from the graph or table, substitute them for *x* and *y* values in the quadratic equation, and then simplify. If the two sides are equal for each set of points, the graph or table matches the equation. The easiest points to pick, if they are integers, are the two *x*-intercepts and the *y*-intercept, but any three points will do.

Substituting three different points into an equation to see if the equation is true can be time consuming. It would really be helpful to get an equation from looking at a graph or a table. You can do just that if you memorize the vertex form of a quadratic equation as given on the right. It's called the "vertex" form because it uses the values for the vertex, *h* and *k*. As long as you can identify the vertex from a graph or a table and have one additional point, you can use this formula to find the equation of the quadratic.

> **"Vertex" Formula for a Quadratic Equation**
>
> $y = a(x - h)^2 + k$

Quadratic Equations From Graphs

Let's start with a graph. Find the equation for the quadratic function graphed below.



This is a three step process:

First, you will need to find the coordinates of the vertex and one other coordinate from the graph. The *y*-intercept is a good one to use. Label the *x*-coordinate of the vertex as ***h*** and the *y*-coordinate ***k*** — you'll see why in a minute. Also, label the coordinates of the other point as *x* and *y*.

vertex = $(\overset{h}{-1}, \overset{k}{-4})$ ***y*-intecept = $(\overset{x}{0}, \overset{y}{-3})$**

Next, you'll need the "vertex" form of a quadratic equation. It may look weird, but it's the quadratic function written in terms of the *x* and *y* values of the vertex (*h*, *k*). Substitute the values you labeled into the equation and solve for *a*. Now, you know two things about your equation: the coefficient of the x^2 term is 1 and the constant term is –3. How do you get the rest of the quadratic? Glad you asked.

$$y = a(x - h)^2 + k$$
$$-3 = a(0 - (-1))^2 + (-4)$$
$$-3 = a(1)^2 - 4$$
$$-3 = a - 4$$
$$1 = a$$

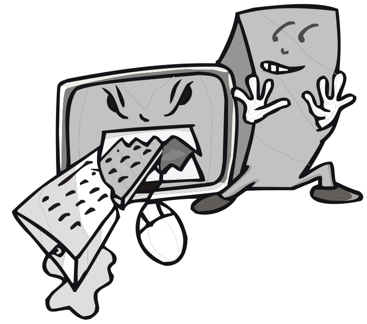
Finally, take the "vertex" form of the equation and substitute the values of *a*, *h*, and *k*. Simplify and you have the quadratic function that matches the graph.

$$y = a(x - h)^2 + k$$
$$y = 1(x - (-1))^2 - 4$$
$$y = (x + 1)^2 - 4$$
$$y = (x + 1)(x + 1) - 4$$
$$y = x^2 + 2x + 1 - 4$$
$$y = x^2 + 2x - 3$$

That wasn't so bad, was it? It would be well worth your time to memorize this "vertex" formula. You'll find it very useful if you need to match a graph to a quadratic equation.

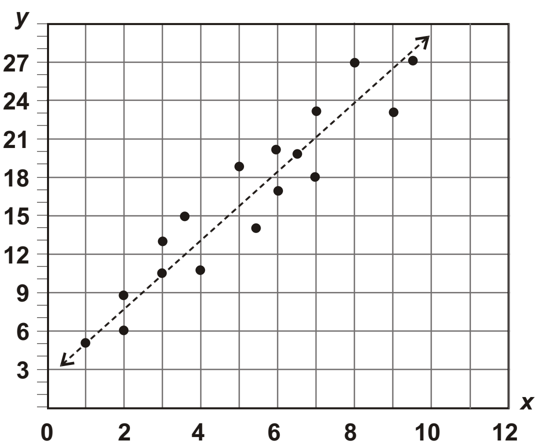
Scatter Plots

Section 27.2
Determining Data Trends



When scatter plots have a definite trend, you can actually draw a line that "fits" the data. If you *eyeball* the relationship, it's called a trend line. If you do a statistical analysis (which is what your calculator does), it's called a **line-of-best fit**. A trend line is just a guess based on observation. The line-of-best fit is a calculation based on statistical analysis. Both are still best guesses, but the line-of-best fit is a better guess

Trend Lines



Finding a trend line by hand is tricky business. But there are a few guidelines you can use to make the attempt a little easier.

You'll need a straight edge — something that won't cover up the data like a string or a stick of spaghetti (uncooked of course; you need a straight line — not a curve).

Try to place the spaghetti, or whatever you're using, on the graph so that there are the same number of points above the line as below the line. Resist the temptation to place the line through both the first and last points. That's usually not a good fit.

It's okay to have points on the line, and you don't count those when you average points above and below the line.

Move the line so that as many points as possible are as close as you can get them to the line. It may take a bit of trial and error to make that happen. When you have it, mark the beginning and end of your spaghetti line and draw the pencil line between the two points.

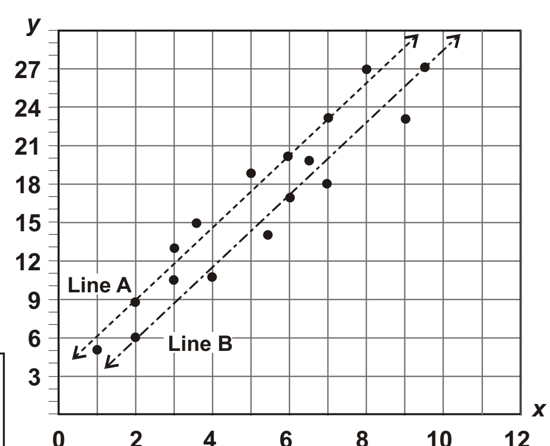
That's your trend line. Check it out to see how well you did. When you count the points above and below the line, you get seven above and seven below. So far, so good. It also looks like you have the distance of the dots above and below the line about equal — some are close and some are not, but all-in-all, they are about the same. That's a good trend line, but it's not the *only* trend line.

Let's say you laid out your spaghetti to look like line A on the graph below. Isn't this a trend line, too? Sure it is. But is it the best one? Count the dots above and below the line. Four above and ten below is not very close to equal, is it?

Look at the dots. The spacing of the dots above and below the line aren't too even either. The dots above the line are a lot closer to the line than the dots below it. So even though it is a trend line, it's probably not the **best** trend for the data.

Line B has the same problem. It's a trend line, but it's just not the best one given the description above.

If you are given choices of which trend line is best, choose according to the following guidelines:



Good Trend Lines

- Same number of points above and below the line
- Points as close to the line as possible
- Equal average distances for points above and below the line