

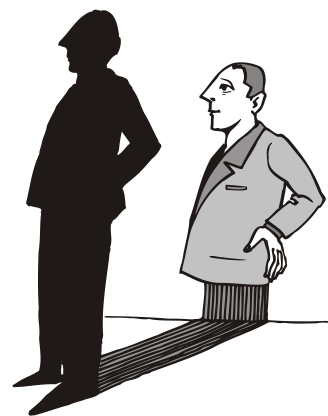
# Exponents and Roots

## Section 3.3 Negative Exponents

One way to think of negative exponents is to rewrite the power as a rational number (fraction) with one as the numerator and the power as the denominator. Or you could say that positive exponents go in the numerator and negative exponents move to the denominator and change their sign to positive. (A negative exponent in the denominator would move to the numerator.)

### Rule for Negative Exponents

$$a^{-m} = \frac{1}{a^m}$$



**Example 1:** Simplify the expression  $(3)^{-3}$ .

Using the rule for negative exponents, the power is moved to the denominator. Then, the exponent can be simplified.

$$3^{-3} \rightarrow \frac{1}{3^3} = \frac{1}{27}$$

If you have a multiplication problem to simplify, move factors with negative exponents into the denominator and keep factors with positive exponents in the numerator. Then you can simplify. Example 2 below shows how.

**Example 2:** Simplify the expression  $3^{-2} \cdot 6^2$ .

Keep the  $6^2$  in the numerator since it has a positive exponent, but move the  $3^{-2}$  to the denominator and change the negative exponent to a positive one. Then do the math.

$$6^2 \cdot 3^{-2} \rightarrow 6^2 \cdot \frac{1}{3^2} = \frac{6^2}{3^2} = \frac{36}{9} = 4$$

shortcut

Now let's look at a couple of division problems. When a division problem is written with a “÷” sign, you can rewrite it as a fraction in two different ways. Choose the way that makes the most sense to you.

**Example 3:** Simplify the expression  $a^{-3} \div a^{-4}$ .

**Step 1:** First, write each factor in fraction form.

$$a^{-4} \div a^{-3} \rightarrow \frac{1}{a^4} \div \frac{1}{a^3}$$

**Step 2:** Remember, dividing by a fraction is the same as multiplying by its inverse.

$$\frac{1}{a^4} \cdot \frac{a^3}{1} = \frac{a^3}{a^4} = \frac{1}{a}$$

**Step 3:** Simplify by using the rules of exponents.

**Example 4:** Simplify the expression  $2a^{-3} \div 3a^{-5}$ .

**Step 1:** The “÷” sign can be replaced with a fraction bar. Rewrite as a fraction before changing the negative exponents. You may see problems written as fractions this way instead of with a “÷” sign.

$$2a^{-3} \div 3a^{-5} \rightarrow \frac{2a^{-3}}{3a^{-5}}$$

**Step 2:** For any variable that has a negative exponent, move it to the other side of the fraction bar and change the exponent to a positive. **Be careful: only move the variables and not the coefficients!**

$$\frac{2\overbrace{a^{-3}}}{3\overbrace{a^{-5}}} \rightarrow \frac{2a^3}{3a^5}$$

**Step 3:** Now simplify using the rules of exponents.

$$\frac{2a^2}{3} \text{ or } \frac{2}{3}a^2$$

# Inequalities

## Section 8.5 Understanding Averages



Working with averages is another real-world situation that may require the use of inequalities. When it comes to averages, the one that students are most aware of is the grades they get for school work. Sounds like a good place to begin.

### Average

$$\frac{\text{sum of item}}{\text{number of items}}$$

You can figure out your grade point average by adding all your test scores and then dividing by the total number of tests. You use this same process to find any average. An average is the sum of items divided by the number of items.

Let's start with finding a current average. Suppose you have taken five tests this semester. Your scores are 88, 75, 91, 77, and 80.

$$\{ 88, 75, 91, 77, 80 \}$$

Calculate your current average by totaling the current grades and dividing by five. Round to the nearest whole number. With these grades, your average is 82.

$$\frac{88 + 75 + 91 + 77 + 80}{5} = 82$$

**Example 1:** You have an A average. Your grades are 90, 95, 92, 97, and 93. There is a test scheduled on Friday before spring break. If you miss the test, you cannot make it up. How much damage can one zero do to your average if your family decides to leave early for spring break?

**Step 1:** Calculate the current average.

$$\frac{90 + 95 + 92 + 97 + 93}{5} = 93$$

**Step 2:** Add one zero and recalculate the average.

$$\frac{90 + 95 + 92 + 97 + 93 + 0}{6} = 78$$

**Step 3:** How much difference did the zero make? Original average minus the new average is fifteen.

$$93 - 78 = 15$$

**One zero made a 15 point difference. You have gone from an A to a C! If there had been fewer grades, the difference would have been even more drastic.**

Now let's say you have a test coming up and you want to know what grade you need to maintain your average. Since you probably don't mind if your average goes up, your calculation becomes an inequality. You want to know the minimum grade, but any grade higher than that will also work!

**Example 2:** Your first five test grades are 88, 75, 91, 77, and 80. What grade do you need to make on the sixth test to maintain your average?

**Step 1:** First, calculate your current average by totaling the current grades, and then divide by five. From above, you've already seen that these grades average to 82.

$$\frac{88 + 75 + 91 + 77 + 80}{5} = 82$$

**Step 2:** To calculate your average after test six, you would add a variable,  $T$ , to stand for the test you haven't taken yet and divide by six.

$$\frac{88 + 75 + 91 + 77 + 80 + T}{6}$$

# Rational Expressions

## Section 11.2 Negative Exponents



So far all the division of monomials you have seen has been with positive exponents. What happens when the rational expression has negative exponents? You pray. Just kidding. Negative exponents aren't really that bad. You just have to pay attention — very careful attention. Let's start with a short review of what you already know about negative exponents.

### Rules for Negative Exponents

$$y^{-7} \rightarrow \frac{1}{y^7}$$

You've already seen that you can make a negative exponent positive by making it the denominator of a fraction. But what if you already have a rational expression and the negative exponents are in the numerator or denominator? You move them. Here's how.

$$\frac{x^{-3}y^2}{y^2} \rightarrow \frac{y^2}{x^3y^2}$$

If the negative exponent is in the numerator, you *move* it to the denominator. If the negative exponent is in the denominator, move it to the numerator. It's just that simple. When you move negative exponents, they become positive.

$$\frac{x^2}{x^3y^{-2}} \rightarrow \frac{x^2y^2}{x^3}$$

$$\frac{2x^{-2}y^{-2}}{3x^{-3}y^{-4}} \rightarrow \frac{2x^3y^4}{3x^2y^2}$$

If you have all negative exponents in the numerator and the denominator, the variables swap places. Make sure you don't swap the coefficients; they already have a positive exponent. They're raised to the power of +1.

### Negative Exponents in Rational Expressions

If a rational expression has negative exponents, use the rules above to make them positive. Once you make the exponents positive, you can simplify the rational expression by canceling common factors. Take a look at these examples.

**Example 1:** Simplify the expression  $\frac{16x^{-3}y^4z^{-2}}{12x^2y^2z^3}$ .

Since there are negative exponents in the numerator, you *move* them to the denominator. Once all the exponents are positive, you can add the exponents that have the same base.

$$\frac{16x^{-3}y^4z^{-2}}{12x^2y^2z^3}$$

**Step 1:** Move the negative exponents to the denominator and make them positive. Remember to move both the base and the exponent. Don't move just the exponent.

$$\frac{16y^4}{12x^2x^3y^2z^3z^2}$$

**Step 2:** Add the exponents with the same bases.

$$\frac{16y^4}{12x^{2+3}y^2z^{3+2}} \rightarrow \frac{16y^4}{12x^5y^2z^5}$$

**Step 3:** Factor the coefficients if you can.

$$\frac{4 \cdot 4y^4}{4 \cdot 3x^5y^2z^5}$$

**Step 4:** Cancel the common factors in the coefficients and use the shortcut to cancel exponents.

$$\frac{4y^2}{3x^5z^5}$$

**Step 5:** After cancelling, regroup what's left.