High School Math: Essential Skills
Student Review Guide

Authors:

Kelly D. Berg
Jerald D. Duncan

Published by Enrichment Plus, LLC

Toll Free: 1-800-745-4706 • Fax 678-445-6702
Web site: www.enrichmentplus.com

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The Authors

Kelly Davis Berg graduated from Clemson University in Clemson, South Carolina, where she earned a Bachelor of Science degree in Chemical Engineering. Besides her background in industrial research, she has worked in the field of educational publishing for over twenty years, and she has taught chemistry laboratory skills to students at the high school and college levels. In addition to her role as project coordinator and executive editor for educational publications in several subject areas, she has also co-authored books in science and mathematics.

Jerald D. Duncan has been involved with education for the past thirty years and has taught math and science at the middle school and high school levels. He was also assistant to the Vocational Director, Cobb County Schools, and the Apprenticeship Coordinator, Cobb County Schools. He is a nationally recognized Curriculum Development facilitator and teacher trainer. He has conducted more than 40 teacher training workshops in over a dozen states in the areas of Applied Pre-Algebra, Academic and Vocational Integration, Cooperative Learning, and Reading Across the Curriculum. He is also a CORD certified trainer in the areas of Applied Math, CORD Algebra, CORD Geometry and Principles of Technology. Jerald is a frequent presenter at the SREB summer conferences and has presented at the Regional NCTM Conference. He is a graduate of Emmanuel College in Franklin Springs, Georgia, and Georgia State University in Atlanta, Georgia.

Simple Order of Operations

Section 2.3

Exponents

Addition, subtraction, multiplication, and division are the four most basic math operations, but they aren't the only ones. Let's add exponents to this mix. What is the correct order of operations for a math problem that includes exponents? To take it a step further, what happens if the problem also includes grouping symbols such as parentheses, absolute value symbols, or a fraction bar? The rules for correct order of operations account for all of these, and the order is given below.

1. **Grouping Symbols**
2. **Exponents (and Radicals)**
3. **Multiplication & Division**
4. **Addition & Subtraction**

To help you remember the order, make up your own sentence with the first letters of the list or remember this one:

Greatly Excuse My Dear Aunt Sally

Grouping Symbols **Exponents** **Multiplication & Division** **Addition & Subtraction**

Exponent Review

First, let's review exponents. (Grouping symbols will be next.)

As you probably remember, addition has a shortcut called *multiplication*. Rather than adding 4 + 4 + 4, you can write 4 × 3. Multiplication also has a shortcut, and it's called using ***exponents*** or ***powers***.

Exponent (or Power)

4^2

Base

$4^2 = 4 \cdot 4 = 16$

$4^3 = 4 \cdot 4 \cdot 4 = 64$

You can rewrite 4 × 4 by using a ***base*** and an ***exponent***. 4^2 means two fours multiplied together.

The ***base*** is the *number being multiplied,* and the ***exponent*** indicates *how many times*. When a number is followed by an exponent, it is said to be "raised to a power."

The number $\mathbf{4^3}$ would mean three fours multiplied together as shown.

Naming a number raised to a power is fairly easy. There are two common ones to remember, and the rest are read base to whatever the exponent indicates.

Any base raised to the second power is said to be ***squared***. Any base raised to the third power is said to be ***cubed***.

As you can see $\mathbf{2^2}$ is read **2 squared**. $\mathbf{3^3}$ is read as **3 cubed**.

The rest, like $\mathbf{2^4}$**,** would be read two to the fourth power or simply two to the fourth, $\mathbf{2^5}$ would be read two to the fifth, and so on.

2^2 = 2 squared

3^3 = 3 cubed

2^4 = 2 to the 4th

2^5 = 2 to the 5th

Section 2.3, continued
Exponents

When you raise a positive number to a power, the answer is also positive, but what happens to a negative number that is raised to a power? The rules are simple. A negative number raised to an even power is positive. A negative number raised to an odd power is negative. If you think about exponents in terms of being a shortcut for multiplication, you can see why these rules work.

Rule #1

A negative number raised to an even power is positive.

$(-2)^2 = -2 \cdot -2 = 4$

Rule #2

A negative number raised to an odd power is negative.

$(-2)^3 = -2 \cdot -2 \cdot -2 = -8$

Here are a few other exponent rules. Any number raised to a power of 1 is equal to that number. Any number raised to a power of 0 is equal to 1.

Exponent of 1

Any number raised to a power of one is equal to that number.

$2^1 = 2$ $(-2)^1 = -2$

Exponent of 0

Any number raised to a power of zero is equal to one.

$2^0 = 1$ $(-2)^0 = 1$

Practice

Simplify the following problems that contain exponents. Be sure you include the correct sign. Write your answers in the blanks.

1. 7^2 ________	2. $(-3)^2$ ________	3. $(-5)^2$ ________	4. 2^3 ________
5. $(-1)^2$ ________	6. $(-1)^3$ ________	7. 6^0 ________	8. $(-4)^1$ ________
9. $(-4)^2$ ________	10. $(-3)^3$ ________	11. 5^2 ________	12. 6^2 ________

Algebra Basics

Section 3.2

Basic Properties of Pre-Algebra

There are certain properties of mathematics that allow you to manipulate and simplify expressions and equations. You may not know their names, but you need to know how to use them. It's a little like recognizing the faces of people you've met without being able to call them by name. It's time to put the "names with the faces," so to speak, and make sure you know what you can do with these basic properties. You should know that these aren't all of the basic properties, but you'll add a few more later in this section.

Commutative Properties of Addition and Multiplication

The commutative properties simply say that numbers can be added in any order or multiplied in any order. If you use symbols (*a* and *b*) instead of numbers, you get universal formulas.

Commutative Properties

For all numbers ***a*** & ***b***

Addition	Multiplication
$a + b = b + a$	$a \cdot b = b \cdot a$

Examples:

$2 + 5 = 5 + 2$ $\quad$ $2 \cdot 5 = 5 \cdot 2$

$x + 1 = 1 + x$ $\quad$ $x \cdot 2 = 2 \cdot x$

Associative Properties of Addition and Multiplication

The associative properties allow you to regroup numbers that are added together or multiplied together.

Associative Properties

For all numbers ***a***, ***b*** and ***c***

Addition	Multiplication
$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Examples:

$(2 + 5) + 1 = 2 + (5 + 1)$ $\quad$ $(2 \cdot 5) \cdot 3 = 2 \cdot (5 \cdot 3)$

$(x + 2) + 1 = x + (2 + 1)$ $\quad$ $(x \cdot 2) \cdot 3 = x \cdot (2 \cdot 3)$

Addition and Subtraction Properties of Zero

The addition and subtraction properties of zero say that you can add zero to any number or subtract zero from any number without changing the number.

Addition and Subtraction Properties of Zero

For any number ***a***

Addition	Subtraction
$a + 0 = a$	$a - 0 = a$

Examples:

$3 + 0 = 3$ $\quad$ $3 - 0 = 3$

$2x + 0 = 2x$ $\quad$ $2x - 0 = 2x$

Multiplication and Division Properties of One

The multiplication and division properties of one allow you to multiply or divide any number by one without changing the number.

Multiplication and Division Properties of One

For any number ***a***

Multiplication	Division
$a \cdot 1 = a$	$a \div 1 = a$

Examples:

$5 \cdot 1 = 5$ $\quad$ $5 \div 1 = 5$

$3x \cdot 1 = 3x$ $\quad$ $3x \div 1 = 3x$

Multiplication and Division Properties of Zero

You can multiply zero by any number or divide zero by any number, and the answer will be zero. Notice that the division property doesn't say that you can divide any number by zero. Division by zero is not permitted. The solution to any number divided by zero is ***undefined***.

Multiplication Property of Zero

For all real numbers ***a***

$$\mathbf{0 \bullet a = 0}$$

Division Property of Zero

For all real numbers ***a*** with $a \neq 0$

$$\mathbf{0 \div a = 0}$$

Examples:

$0 \cdot 4 = 0$

$0 \cdot x = 0$

$0 \div 3 = 0$

$0 \div 2x = 0$

$2x \div 0 =$ undefined

Practice

Review the basic properties given in this sub-section. Based on these properties, determine if the following mathematical statements are equal or not. If both sides of the equation are equal, write an *E* in the blank. If the sides are not equal, write an *N* in the blank.

Examples: __E__ $5 \bullet x = x \bullet 5$ __N__ $5 \div x = x \div 5$

________ 1. $x + 2 = 2 + x$

________ 2. $0 \bullet 4x = 4x$

________ 3. $(7 \bullet 4) \bullet 2 = 7 \bullet (4 \bullet 2)$

________ 4. $(x + 4) + 5 = x + (4 + 5)$

________ 5. $8 - 0 = 0 - 8$

________ 6. $x - 0 = 0$ (if $x \neq 0$)

________ 7. $1 \bullet 2x = 2x$

________ 8. $x + 0 = 0$ (if $x \neq 0$)

________ 9. $3 \div 4 = 4 \div 3$

________ 10. $x \bullet 1 = 1$

________ 11. $2 \bullet (3 \bullet x) = (2 \bullet 3) \bullet x$

________ 12. $x \div 1 = x$

________ 13. $3x \div 0 = 0$

________ 14. $x + 0 = x$ (if $x \neq 0$)

________ 15. $0 \div 4 = 0$

________ 16. $2x \div 1 = 1$

________ 17. $(7 \bullet x) \div 4 = 7 \bullet (x \div 4)$

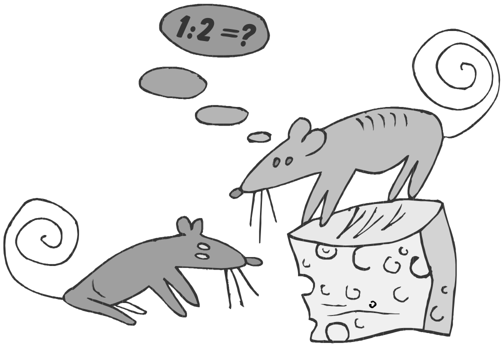
________ 18. $2 \bullet x = x \bullet 2$

________ 19. $5x - 0 = 5x$

________ 20. $4 - x = x - 4$

Ratios and Proportions

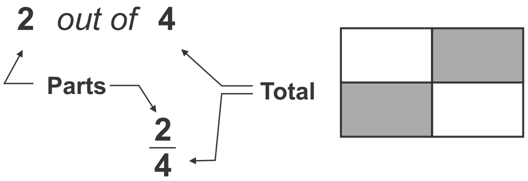
Section 5.1
Ratios



A **ratio** is a comparison of two numbers by division. Ratios come in several different forms, some of which are not so easy to recognize. Two forms are actually division operations: the fraction bar and the division symbol. These are the easy ones. The other three are not so easy. They use other forms of comparison that don't look like division.

$\frac{1}{3}$	1 ÷ 3	1 to 3	1 : 3	1 out of 3
Comparison using the fraction bar	**Comparison using the division symbol**	**Comparison using the word "to"**	**Comparison using the colon symbol**	**Comparison using words to show parts of a whole**

Ratios can compare parts of a whole, or they can compare one quantity to another quantity. When a ratio compares parts of a whole, the numerator (top number in the fraction) is the selected parts, and the denominator (bottom number in the fraction) is the total parts that make up the whole. The ratio becomes parts over total.



Lowest Terms

Ratios are expressed as fractions in lowest terms. However, real world situations don't usually result in fractions in lowest terms. You remember lowest terms. You find the largest factor that is common to both numerator and denominator, and divide both by that factor.

Example 1: **Convert the ratio $\frac{12}{48}$ into lowest terms.**

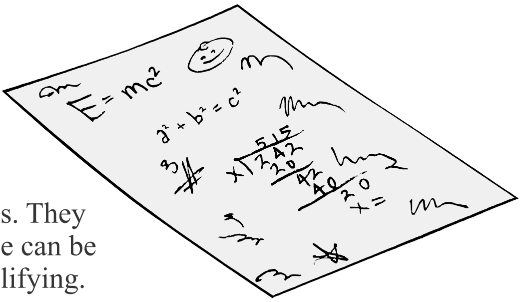
12 and 48 have several common factors. You can probably see that 2, 4, and 6 are all common factors, but you may not realize that none of these are the greatest common factor (GCF). The GCF is 12. If you do not choose the greatest common factor to reduce a fraction, the resulting fraction will not be in lowest terms. That's okay. Just reduce again until you can reduce no more!

① GCF is 12	First identify the greatest common factor.
② $\frac{\cancel{12}\cdot 1}{\cancel{12}\cdot 4}$ or $\frac{\overset{1}{\cancel{12}}}{\underset{4}{\cancel{48}}}$	Then divide the numerator and the denominator by that common factor. You can write out the factors and cancel, or you can use a shorthand method to show division.
③ $\frac{1}{4}$	The resulting ratio is in lowest terms.

Now let's see how ratios can be used.

Algebra Word Problems

Section 7.5
Introduction to Formulas



Formulas are special equations that show a relationship between quantities. They have multiple variables that represent the different quantities. One variable can be solved by substituting known values for the other variables and then simplifying.

Sometimes a word problem may give you a formula to use. But other times you will need to identify the correct formula to use (which may be found on a formula sheet). If you do not recognize the correct formula, you may not be able to answer the question.

Two common formulas are the distance formula and the simple interest formula. Other common formulas are found in geometric relationships, but you'll see those later in Section 8.

Distance Formula

distance = rate × time

$d = r \cdot t$ or $d = rt$

Simple Interest Formula

interest = principal × rate × time

$i = P \cdot r \cdot t$ or $i = Prt$

As you can see in the formulas above, the "variables" such as *d*, *r*, and *t* in the distance formula and *i*, *p*, *r*, and *t* in the simple interest formula represent specific quantities. Also notice that when variables are multiplied together, no multiplication sign is needed. The term "*rt*" means "*r* times *t*" and the term "*prt*" means "*p* times *r* times *t*."

Rearranging Formulas

Look at the distance formula. If you know the rate, *r*, and the time, *t*, you can substitute in those values to find the distance, *d*. But, what if you know the distance and the time but need to find the rate? The formula would need to be "rearranged" to solve for rate. You could rearrange the formula mathematically, or you could use the "Thumb/Circle Shortcut." Both of these methods are shown below.

To rearrange the formula mathematically, you would isolate the variable you want by algebraically solving the formula for a different variable. Let's say you want to solve for rate in the distance formula.

distance = rate × time

$$d = rt$$

$$\frac{d}{t} = \frac{rt}{t}$$

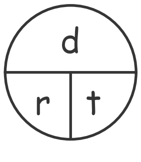
$$\frac{d}{t} = \frac{r\cancel{t}}{\cancel{t}}$$

$$\frac{d}{t} = r \text{ or } r = \frac{d}{t}$$

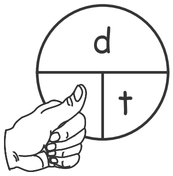
To isolate the rate on one side of the equation, use the multiplication principle. Since you can do anything to an equation as long as you do it to both sides, you can divide both sides by *t*. The *t*'s on the right side of the equation cancel, and you are left with an equation for *r* in terms of *d* and *t*.

To use the Thumb/Circle Shortcut, you draw a circle like the one below and divide it as shown. The top of the circle represents the left side of the equation, and the bottom of the circle represents the right side.

$d = rt$



To rearrange for a different quantity, you simply put your thumb over the variable you want to solve for and write down the relationship of the other two variables.



To solve for rate, *r*, put your thumb over the *r*. Then write the remaining relationship as shown by the circle.

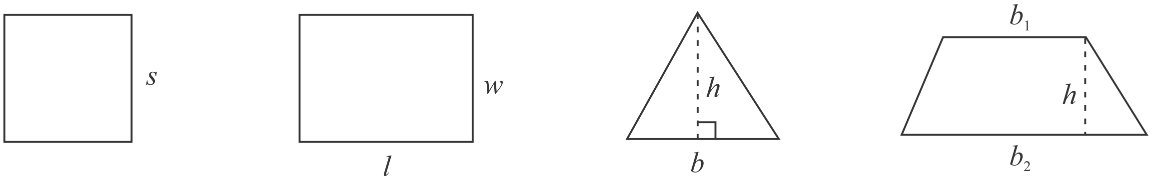
$$r = \frac{d}{t}$$

Two Dimensional Geometric Formulas

Section 8.2
Area of Polygons

Area is the amount of space inside a figure. It is measured in square units such as inches squared (in^2), feet squared (ft^2), etc. Finding the area of a polygon always involves multiplication of two dimensions. The formulas for the most common polygons are given in the chart below.

Area Formulas



square	rectangle	triangle	trapezoid
$A = s^2$	$A = lw$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(b_1 + b_2)$

CAREFUL: When using any of these formulas, be sure that the units are the same. Area is given in square units such as square inches or square feet. The only way to get square inches is to multiply inches by inches. To get square feet, you must multiply feet by feet. You cannot multiply inches by feet without first converting one of the units to match the other.

The most common English conversion for length is feet to inches or inches to feet. Remember that 1 foot equals 12 inches! To convert feet to inches, multiply by 12. To convert inches to feet, divide by 12. Practice doing the following conversions.

1 foot = 12 inches

Practice 1

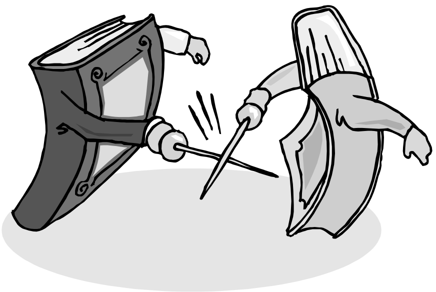
Perform the following unit conversions from feet to inches or from inches to feet. When converting from inches to feet, record the answer as a fraction that is reduced to lowest terms.

1. 2 feet = ________ inches
2. 3 feet = ________ inches
3. 4 inches = ________ feet
4. 6 inches = ________ feet
5. 3 inches = ________ feet
6. 9 inches = ________ feet

Multiplying Polynomials

Section 12.2

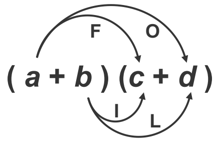
Multiplying Binomials Using FOIL



Now the real fun begins! Next, let's multiply two binomials.

FOIL

When two binomials are to be multiplied, each term of the first binomial must be multiplied by each term of the second binomial.



$ac + ad + bc + bd$

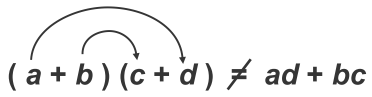
As you consider this method, take special notice of two things:

1. The number of terms in the product is the same as the number of terms in both binomials. (2 terms × 2 terms = 4 terms)
2. There is a pattern to multiplying the terms. It's called FOIL.

F	O	I	L
First Terms	Outside Terms	Inside Terms	Last Terms

DO NOT make the common mistake of simply multiplying the first terms and the last terms. The answer will be WRONG.

WRONG!



Example 1: Simplify (x + 1)(x + 2)

Multiplying two binomials is a five-step process. Four of the steps use the FOIL method of multiplying each term. The fifth step is to combine like terms.

F	O	I	L
x^2	$2x$	x	2
① $(x + 1)(x + 2)$	② $(x + 1)(x + 2)$	③ $(x + 1)(x + 2)$	④ $(x + 1)(x + 2)$
Multiply the first terms of each binomial.	Multiply the outside terms of each binomial.	Multiply the inside terms of each binomial.	Multiply the last terms of each binomial.

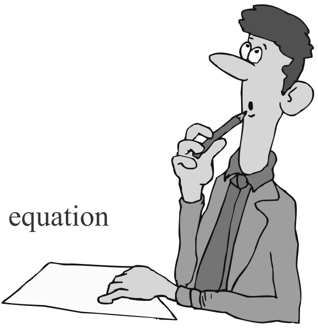
$$x^2 + \underbrace{2x + x}_{\downarrow} + 2$$

⑤ $x^2 + 3x + 2$ Combine like terms.

Graphing Linear Equations

Section 16.6

Slope-Intercept Form of a Line



A linear equation contains an *x* and/or a *y*, and those variables can be found anywhere in the equation (except in the denominator of a fraction.) Look at the linear equations given below. Do you notice anything special about them?

$2y - 2x = 4$ $\qquad$ $2x = 2y - 4$ $\qquad$ $2y = 2x + 4$ $\qquad$ $y = x + 2$

The equations above are actually the same equation written in different ways. If you graphed each of these equations, they would represent exactly the same line.

Slope-Intercept Form of a Line

Is there an advantage to rewriting an equation of a line in any particular form? Actually, there is. Something interesting happens when you solve a linear equation for *y* in terms of *x*. In this form, the coefficient in front of the *x* represents the slope of the line, and the constant term represents the *y*-intercept. This form is called the **slope-intercept form of a line**. Can you see why this is a good thing? The equation of a line tells you the slope and the *y*-intercept. With these two pieces of information, you can easily graph the line!

Slope-Intercept Form

$$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$$

If you happen to see an equation written as

$$f(x) = mx + b$$

don't panic! $f(x)$ means the same thing as *y*. The rest of the equation is exactly the same.

Example 1: **Identify the slope and the y-intercept of the line y = –2x + 1.**

$$y = \underbrace{-2}_{\text{slope is } -2}x + \underbrace{1}_{\substack{y\text{-intercept is } 1 \\ (0,\ 1)}}$$

This equation is in slope-intercept form. The coefficient in front of the *x* is the slope, so the slope is –2. The constant term gives the *y*-intercept, which in this example is 1. Remember that a *y*-intercept at 1 is represented by the point (0, 1).

Example 2: **Identify the slope and the y-intercept of the line $f(x) = \frac{1}{2}x - 3$.**

The "$f(x)$" means the same thing as "*y*." You identify slope and *y*-intercept the same as before.

$$f(x) = \underbrace{\tfrac{1}{2}}_{\text{slope is } \frac{1}{2}}x \underbrace{-\ 3}_{\substack{y\text{-intercept is } -3 \\ (0,\ -3)}}$$

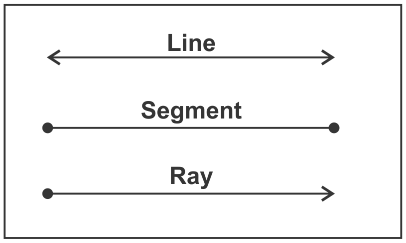
This equation is also in slope-intercept form. The slope of this line is one-half. The *y*-intercept is –3. When the constant term is subtracted, the *y*-intercept is negative.

Angles and Triangles

Section 20.1
Introducing Angles

As you have already seen, lines can either intersect or be parallel. Intersecting lines form angles, and those angles have mathematical relationships. Parallel lines by themselves may not be very interesting, but when they are intersected by another line, you have more angles and angle relationships. First, let's review some basics about lines and angles.

Lines, Segments, and Rays

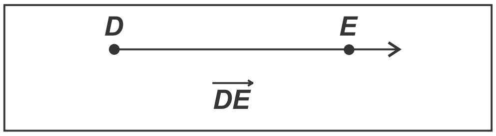


A **line** extends endlessly in both directions. It is drawn with arrows on the ends to show that it extends indefinitely.

A **segment** is the part of a line between two points represented by closed dots on both ends.

Now, what if you had an endpoint on one end and an arrow on the other? The line would start at a definite point and extends indefinitely in the other direction. That type of line is called a **ray**.

When naming rays, you identify the endpoint and one other point on the ray. The capital letters of the points are written with the ray symbol above. ***NOTE:*** *Always name the endpoint first when naming rays.*

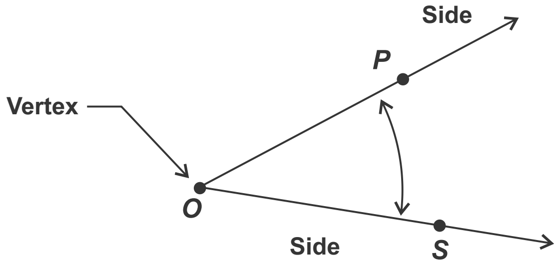


Angles

Angles are sometimes defined as ***two rays with a common endpoint.*** According to that definition, ray ***OP*** and ray ***OS*** to the right form an angle.

Angles are named using the points that identify the two rays listed in either direction with the common endpoint as the middle letter. This angle would be angle ***POS*** or angle ***SOP***, depending on which direction you go.

To avoid having to write the word *angle*, you can use the angle symbol ($\angle$).



$\angle POS$ or $\angle SOP$

The two rays that make up an angle are called sides, and the common endpoint is called the vertex. The curved distance between the sides is the angle measure. Angles are measured in degrees by using a protractor. To indicate the measure of an angle, a lower case *m* is used with the angle symbol. The measure of angle *POS* is written as $m\angle POS$.

To make things a little more convenient, angles can also be named using letters, numbers, or even letters of the Greek alphabet.

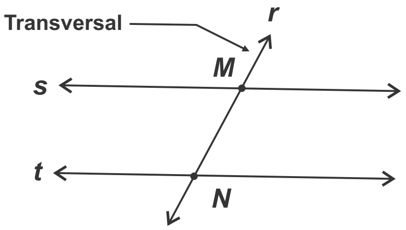
$\angle POS$ or $\angle SOP$ $\quad$ $\angle 1$ $\quad$ $\angle A$ or $\angle a$ $\quad$ $\angle \alpha$

$m\angle 1$ means *the measure of angle 1*

Angles and Triangles

Section 20.6
Parallel Lines Intersected by a Transversal

You've already seen that two lines that do not intersect are called parallel lines. But what if the parallel lines are intersected by a third line?



The line that intersects, or cuts across, a pair of parallel lines to form two intersections is called a **transversal**.

Line ***s*** is parallel to line ***t***, or ***s*** || ***t***. The symbol (||) indicates that lines are parallel. Line ***r*** is a transversal of the parallel lines *s* and *t*. It intersects the lines at points ***M*** and ***N***, respectively.

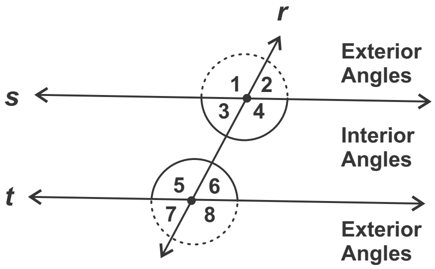
When a transversal intersects a pair of parallel lines, eight angles are formed — 4 pairs of vertical angles. You already know that the angle measures of each pair of vertical angles are equal. But there are other angle relationships when you consider both intersections together. These angles have special names and specific relationships that you will need to remember.

Exterior Angles

∠1, **∠2**, **∠7**, and **∠8** are called ***exterior*** angles because they are formed "outside" the pair of parallel lines.

Interior Angles

∠3, **∠4**, **∠5**, and **∠6** are called ***interior*** angles because they are formed "inside" the pair of parallel lines.



Exterior and interior angles are also paired more specifically. These special pairs have specific relationships. The special angle pairs are called alternate interior angles, alternate exterior angles, corresponding angles, same side interior angles, and same side exterior angles.

Alternate Interior Angles

Alternate interior angles must meet three conditions:

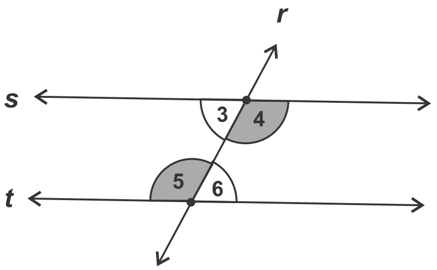
1. They must be **inside the parallel lines** (*interior* angles).
2. They must be on **opposite sides** of the transversal.
3. They must have **different vertices** (plural of vertex).

The four interior angles shown to the right are **∠3**, **∠4**, **∠5**, and **∠6**. One pair of alternate interior angles is **∠3** and **∠6** since they are on opposite sides of the transversal and have different vertices. The other pair is **∠4** and **∠5**.

The most important thing to remember about alternate interior angles is that they are equal. (Remember, the transversal must cut across parallel lines.) So *m*∠**3** = *m*∠**6** and *m*∠**4** = *m*∠**5**.

If *m*∠**3** = 30º, what does *m*∠**6** equal? Did you say 30º? You are right!

Alternate Interior Angles are Equal.



***m*∠3 = *m*∠6**
***m*∠4 = *m*∠5**

Section 20.6, continued
Parallel Lines Intersected by a Transversal

Alternate Exterior Angles

Alternate exterior angles must also meet three conditions:

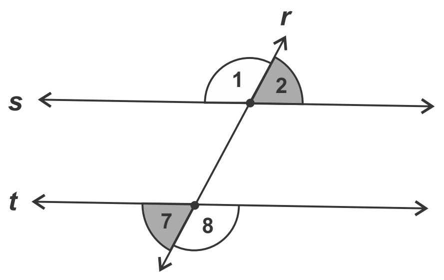
1. They must be **outside the lines** (*exterior* angles).
2. They must be on **opposite sides** of the transversal.
3. They must have **different vertices**.

One pair of alternate exterior angles is $\angle \mathbf{1}$ and $\angle \mathbf{8}$. The other pair is $\angle \mathbf{2}$ and $\angle 7$.

Alternate exterior angles are also equal. $m\angle \mathbf{1} = m\angle \mathbf{8}$ and $m\angle \mathbf{2} = m\angle 7$.

If $m\angle \mathbf{1} = 150^\circ$, what does $m\angle \mathbf{8}$ equal? You're right if you said 150°!

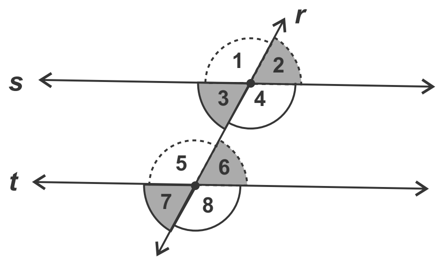
Alternate Exterior Angles are Equal.



$m\angle 1 = m\angle 8$

$m\angle 2 = m\angle 7$

Corresponding Angles are Equal.



$m\angle 1 = m\angle 5$ $\quad$ $m\angle 3 = m\angle 7$

$m\angle 2 = m\angle 6$ $\quad$ $m\angle 4 = m\angle 8$

Corresponding Angles

Another relationship between pairs of angles is called corresponding angles. To be corresponding angles, the pair must be on the same side of the transversal and in the same position on the different parallel lines. Four pairs of angles fit that description:

$\angle \mathbf{1}$ and $\angle \mathbf{5}$ $\quad$ $\angle \mathbf{3}$ and $\angle 7$

$\angle \mathbf{2}$ and $\angle \mathbf{6}$ $\quad$ $\angle \mathbf{4}$ and $\angle \mathbf{8}$

Corresponding angles are also equal, so in this example $m\angle \mathbf{1} = m\angle \mathbf{5}$, $m\angle \mathbf{2} = m\angle \mathbf{6}$, $m\angle \mathbf{3} = m\angle 7$ and $m\angle \mathbf{4} = m\angle \mathbf{8}$.

There's one more set of conditions you may find helpful when problem-solving with parallel lines. They're called **same side** angles. The angle relationships are *not* equal like you've seen to this point. Instead, they are supplementary.

Same Side Interior Angles

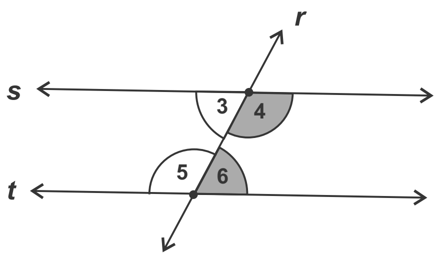
Same side interior angles must only meet two conditions:

1. They must be on the same side of the transversal.
2. They must be inside the parallel lines.

One pair of same side interior angles is $\angle \mathbf{3}$ and $\angle \mathbf{5}$.
The other pair is $\angle \mathbf{4}$ and $\angle \mathbf{6}$.

Same side interior angles are supplementary. That means that $m\angle \mathbf{3} + m\angle \mathbf{5} = \mathbf{180^\circ}$ and $m\angle \mathbf{4} + m\angle \mathbf{6} = \mathbf{180^\circ}$.

Same Side Interior Angles are Supplementary.

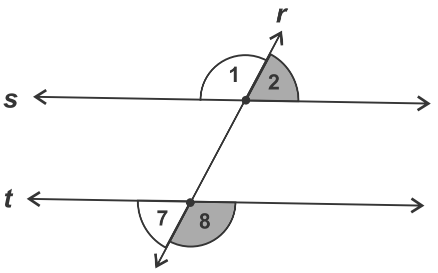


$m\angle 3 + m\angle 5 = 180^\circ$

$m\angle 4 + m\angle 6 = 180^\circ$

Section 20.6, continued
Parallel Lines Intersected by a Transversal

Same Side Exterior Angles are Supplementary.



$m\angle 1 + m\angle 7 = 180°$

$m\angle 2 + m\angle 8 = 180°$

Same Side Exterior Angles

Same side exterior angles must also meet two conditions:

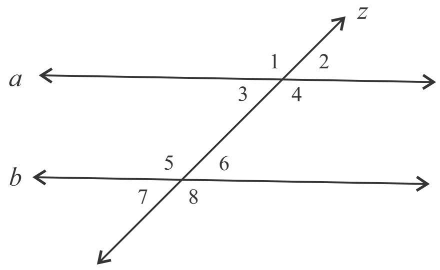
1. They must be on the same side of the transversal.
2. They must be outside the parallel lines.

One pair of same side exterior angles is ∠**1** and ∠**7**.
The other pair is ∠**2** and ∠**8**.

Same side exterior angles are supplementary. That means **$m\angle 1 + m\angle 7 = 180°$** and **$m\angle 2 + m\angle 8 = 180°$**.

Practice 1

Use the diagram given below to answer the following questions. Write your answers in the spaces provided.



______ 1. As alternate exterior angles, ∠1 is paired with ___.

______ 2. As alternate interior angles, ∠3 is paired with ___.

______ 3. The corresponding angle to ∠7 is ___.

______ 4. The measures of interior angle 4 and exterior angle 1 are equal. The measure of angle 4 is also equal to the measure of which corresponding angle?

______ 5. Exterior angles 1 and 2 are supplementary. Angle 2 is also supplementary to which other exterior angle?

______ 6. Interior angle 4 is supplementary to angle 3. Angle 4 is also supplementary to which other interior angle?

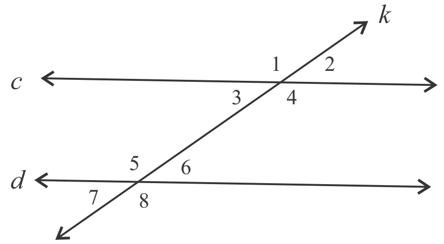
______ 7. Angle 3 is an interior angle. The measure of angle 3 is equal to the measure of which other interior angle?

______ 8. Angle 7 is an exterior angle. The measure of angle 7 is equal to the measure of which other exterior angle.

When two parallel lines are cut by a transversal, all of the angle measures are related. If you know the measure of one of the angles, you can use the relationships you've just seen to determine the measures of all the other angles. You can also use your knowledge of supplementary angles and vertical angles. To be sure you understand these relationships, fill in the angle measures in the diagram below.

Practice 2

In the diagram below, line *c* is parallel to line *d*. The measure of angle 1 is given. Determine the measures of all the other angles. Write your answers in the blanks provided.



$m\angle 1 = 145°$

$m\angle 2 =$ ________

$m\angle 3 =$ ________

$m\angle 4 =$ ________

$m\angle 5 =$ ________

$m\angle 6 =$ ________

$m\angle 7 =$ ________

$m\angle 8 =$ ________

Probability

Section 24.1
Introducing Probability

Probability can be defined as how likely an event is to occur. It has its own vocabulary. You will need to be familiar with some terms in order to make sense of probability. Here's a summary.

Probability Vocabulary

Outcome – The result of a single trial

Event – Any outcome or group of outcomes

Sample Space – All the possible outcomes

Favorable Outcome – The outcome you wanted

Theoretical Probability – Favorable outcomes divided by possible outcomes.

$$\frac{\text{Favorable}}{\text{Possible}}$$

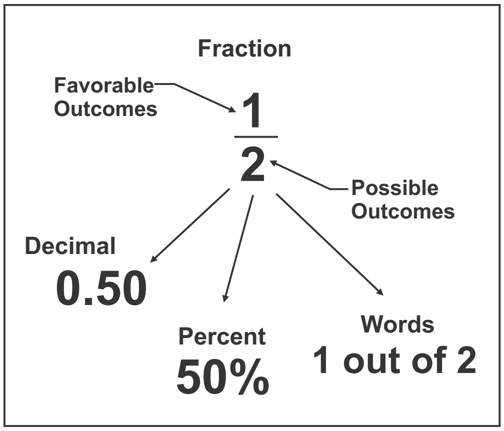
It may sound complicated, but it will make more sense as you see how it works.

Probability of an event is a value from 0 to 1, expressed as a fraction, a decimal, or a percent. A probability of ***zero*** means it is ***impossible*** for the event to occur. A probability of ***one*** means the event is ***certain*** to occur.

Probability of a Coin Toss

When you flip a coin, there are two possible outcomes. The sample space is a head or a tail. (There is an extremely slim chance that the coin could land on edge, so that probability is near zero.)

For all practical purposes, there are two possible outcomes. The **theoretical probability** of getting a heads is ***favorable divided by total.***



Probability as a fraction is read as 1 out of 2. When probability is given as a fraction, the fraction must be reduced to lowest terms. If you do the math, you can get probability as a decimal: 0.50. Converted to a percent, this probability is 50%.

If you actually tossed a coin 1,000 times, recorded each trial, and tallied the outcomes, you would be very close to the theoretical probability of 50% heads and 50% tails. In fact, the more times you repeat the experiment, the closer the outcomes will match the 50–50 probability.

The actual outcomes obtained as the result of performing the trials is called ***empirical or experimental probability***. The formula for experimental probability changes a bit from the theoretical one. The fraction is bigger, and the result may be close but not exactly the same as the theoretical probability.

Empirical or Experimental Formula

$$\frac{\text{Favorable Outcomes}}{\text{Number of Trials}}$$

Practice Test 1

Overview

Introduction
The practice test that follows is provided to help you determine how well you have mastered basic high school math skills.

Directions
Read each question carefully and darken the circle corresponding to your answer choice. Once you have completed this practice test, circle the questions you answered incorrectly on the practice test evaluation chart on page PT1-26. For each question that you missed on the practice test, review the corresponding sections in the book as given in the evaluation chart. Read the instructional material, do the practice exercises, and take the section review test at the end of each section.

1. Simplify the following expression:

$$8x^2 + 5x - 2x^2 + 4x$$

A $6x^2 + 9x$
B $6x^2 + x$
C $10x^2 + x$
D $6x^4 + 9x^2$

Ⓐ Ⓑ Ⓒ Ⓓ

2. Simplify the following expression:

$$2x + y - (x - 2y)$$

A $x - y$
B $x + y$
C $x + 3y$
D $3x - y$

Ⓐ Ⓑ Ⓒ Ⓓ

3. Solve the following algebraic equation:

$$2x + 5 = 6x - 7$$

A -3
B $-\frac{1}{2}$
C $\frac{3}{2}$
D 3

Ⓐ Ⓑ Ⓒ Ⓓ

4. Solve the following inequality:

$$x + 4 < 3x - 8$$

A $x > 3$
B $x < 3$
C $x > 6$
D $x < 6$

Ⓐ Ⓑ Ⓒ Ⓓ

5. Simplify the following expression:

$$4 - 8 \div (3 - 5)^2 + 2$$

A 1
B 3
C 4
D 10

Ⓐ Ⓑ Ⓒ Ⓓ

6. Simplify the following expression:

$$(3x + 2)^2$$

A $3x^2 + 2$
B $9x^2 + 4$
C $6x^2 + 12x + 4$
D $9x^2 + 12x + 4$

Ⓐ Ⓑ Ⓒ Ⓓ

7. Solve the following quadratic equation:

$$4x^2 - 1 = 0$$

A $\frac{1}{2}, -\frac{1}{2}$
B $\frac{1}{4}, -\frac{1}{4}$
C $0, 4$
D $2, -2$

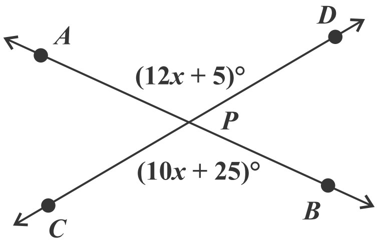
Ⓐ Ⓑ Ⓒ Ⓓ

8. What is the range of the function $g(x) = 3x - 1$ when the domain is $\{-1, 2, 5\}$?

A $\{0, 1, 2\}$
B $\{1, 2, 3\}$
C $\{-4, 5, 14\}$
D $\{-1, 2, 5\}$

Ⓐ Ⓑ Ⓒ Ⓓ

37. **Lines *AB* and *CD* intersect at point *P*. What is the measure of ∠*APD*?**



A 10°
B 30°
C 110°
D 125°

Ⓐ Ⓑ Ⓒ Ⓓ

38. **Solve the following algebraic equation:**

$$4x + 1 = 11$$

A $\frac{5}{2}$

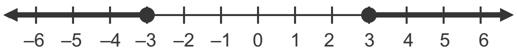
B 3

C 40

D 48

Ⓐ Ⓑ Ⓒ Ⓓ

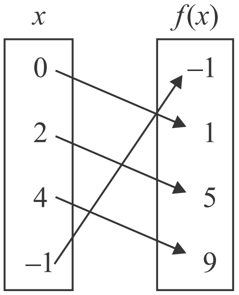
39. **Which inequality describes the graph below?**



A $-3 \leq x < 3$
B $x \leq -3$ and $x \leq 3$
C $x \geq -3$ or $x \leq 3$
D $x \leq -3$ or $x \geq 3$

Ⓐ Ⓑ Ⓒ Ⓓ

40. **Which function describes the mapping below?**



A $f(x) = x - 1$
B $f(x) = x + 1$
C $f(x) = 2x - 1$
D $f(x) = 2x + 1$

Ⓐ Ⓑ Ⓒ Ⓓ

41. **What is the solution of the following system of linear equations?**

$y = 2x$
$x - y = 8$

A (–8, –16)
B (–8, 0)
C (0, –8)
D (16, –8)

Ⓐ Ⓑ Ⓒ Ⓓ

42. **A box has a volume of 2016 cubic inches. Its height is 14 inches, and it has a square base. What is the length of a side of the base?**

A 12 inches
B 24 inches
C 72 inches
D 144 inches

Ⓐ Ⓑ Ⓒ Ⓓ

Practice Test 1

Evaluation Chart

Note: ***Section 1*** covers basic skills that should be reviewed first before going on to the more specific skills listed for each question below.

If you missed question #:	Go to section(s):	If you missed question #:	Go to section(s):	If you missed question #:	Go to section(s):
1	3.1, 3.2, 3.3, 11.1	35	6.4, 6.6	69	13.1, 13.4
2	3.1, 3.2, 3.3, 3.5, 3.8, 11.1, 11.2	36	16.2, 18.3, 18.4	70	7.1, 7.2, 19.3, 19.4, 19.5, 19.6
3	3.3, 4.1, 4.2, 4.4, 4.5, 4.6	37	20.1, 20.2, 20.3, 20.5	71	6.1, 6.4, 6.5, 6.6
4	6.1, 6.3	38	4.1, 4.2, 4.3, 4.4	72	15.1, 15.2, 15.3
5	2.1, 2.2, 2.3, 2.4	39	6.4, 6.6, 6.7	73	20.1, 20.2, 20.3, 20.8, 20.9, 20.10
6	12.2, 12.3	40	17.2, 17.3, 17.4, 18.1, 18.6	74	20.1, 20.2, 22.1, 22.2, 22.3, 22.4
7	13.4, 14.2	41	19.1, 19.3, 19.4, 19.5	75	5.1, 5.2, 5.3, 5.4, 5.5
8	18.1, 18.2	42	8.5, 9.1, 14.2, 14.5, 14.6	76	7.1
9	3.1, 3.2, 3.3, 3.4, 3.5, 3.8, 11.1, 11.2	43	20.1, 20.2, 22.1, 22.2, 22.3, 22.4	77	5.1, 5.2, 5.3, 5.4, 5.5
10	4.3, 10.1, 10.2, 10.3	44	6.1, 6.2	78	23.1, 23.6
11	8.1, 9.3	45	6.1, 6.3	79	7.1
12	3.1, 3.3, 3.6, 3.7, 3.8	46	18.1, 18.5	80	7.1, 7.2, 7.4, 7.5, 7.7
13	3.3, 3.5, 3.8, 11.1, 11.3, 11.4, 11.5	47	15.1, 15.4, 15.5	81	18.3, 18.4
14	20.1, 20.2, 20.3, 20.5, 20.6, 20.7	48	15.1, 15.3, 17.3	82	15.1, 15.2, 16.6, 16.7
15	15.1, 15.2, 16.1	49	15.1, 15.2, 16.4, 16.5, 16.6, 16.7	83	15.1, 15.2, 16.6, 16.7
16	15.1, 15.2, 16.2	50	15.1, 15.2, 16.4, 16.5	84	15.1, 15.2, 16.3, 18.3
17	2.1, 2.2, 3.1, 3.2, 3.3, 11.1	51	2.1, 2.2, 2.5	85	16.6, 16.7, 17.3
18	13.1, 13.3	52	13.1, 13.2, 13.6, 13.7	86	24.1, 24.2, 24.3
19	13.1, 13.2, 14.3	53	4.2, 13.1, 13.7, 14.1, 14.5	87	20.1, 20.2, 20.3, 20.5, 20.6, 20.7
20	19.1, 19.3, 19.4, 19.5	54	6.1, 6.3	88	21.1, 21.2, 21.3
21	13.1	55	10.1, 10.2, 10.4	89	6.1, 6.3, 6.4, 6.5, 6.7
22	4.1, 4.2, 4.4, 4.5, 4.6, 5.3	56	13.3, 14.4	90	22.1, 22.2, 22.3
23	18.1, 18.2	57	19.1, 19.3, 19.4, 19.5	91	23.1, 23.4
24	4.1, 4.2, 4.4, 4.5, 5.3	58	18.1	92	5.1, 5.2, 5.3, 5.4, 5.5, 5.7
25	15.1, 15.2, 15.3	59	8.1, 11.7	93	24.1, 24.2
26	24.1, 24.2, 24.5	60	21.1, 21.2	94	21.1, 21.2, 21.3, 21.4
27	18.1, 18.2	61	3.6, 3.7, 8.4, 12.1, 12.5	95	8.2, 8.3
28	7.1, 7.4	62	15.1, 15.2, 16.6, 16.7	96	21.1, 21.2, 21.3, 21.4
29	20.1, 20.2, 22.1, 22.2, 22.3	63	9.1	97	15.1, 15.6
30	23.1, 23.5	64	7.1, 7.2, 7.3	98	23.1, 23.2, 23.3
31	18.1, 18.3	65	24.1, 24.2, 24.4	99	5.1, 5.2, 5.3, 5.4, 5.5, 5.6
32	16.6, 16.7, 19.1, 19.2	66	8.4	100	18.1, 18.4
33	4.3, 12.2, 12.3, 12.4	67	7.1, 7.2, 19.3, 19.4, 19.5, 19.6		
34	8.1	68	6.1, 6.3, 6.4, 6.5		