

# **Algebra 1: The Fundamentals**

## **Student Review Guide**

Author:  
**Jerald D. Duncan**

---

**Published by Enrichment Plus, LLC**

**Toll Free: 1-800-745-4706 • Fax 678-445-6702**  
**Web site: [www.enrichmentplus.com](http://www.enrichmentplus.com)**

# Table of Contents

<b>The Author/Acknowledgments</b>	v	<b>Section 7</b>	
<b>Preface/How to Use This Book</b>	vi	<b>Using Formulas</b>	
<b>Pretest</b>	7	7.1 Perimeter and Circumference Formulas	115
Evaluation Chart	26	7.2 Area and Surface Area Formulas	118
<b>Section 1</b>		7.3 Volume Formulas	121
<b>Algebra Basics</b>		7.4 Square Root Applications in Geometry Word Problems	123
1.1 Classifying Numbers	27	7.5 Using the Pythagorean Theorem	125
1.2 Math and Algebra Vocabulary	31	7.6 Rearranging Formulas	129
1.3 Positive and Negative Numbers	34	7.7 Rearranging Variables in Word Problems	131
1.4 Substitution Principle	37	7.8 Rearranging Formulas to Solve Word Problems	133
Section 1 Review	39	Section 7 Review	135
<b>Section 2</b>		<b>Section 8</b>	
<b>Properties of Real Numbers</b>		<b>Inequalities</b>	
2.1 Basic Properties	41	8.1 Solving Simple Inequalities	137
2.2 Introduction to Order of Operations	45	8.2 Solving Multi-Step Inequalities	140
2.3 Order of Operations with Grouping Symbols	47	8.3 Graphing Inequalities	142
Section 2 Review	51	8.4 Solving Word Problems with Inequalities	145
<b>Section 3</b>		8.5 Understanding Averages	149
<b>Exponents and Roots</b>		Section 8 Review	152
3.1 Multiplying and Dividing with Exponents	53	<b>Section 9</b>	
3.2 Powers Raised to Powers	55	<b>Absolute Value</b>	
3.3 Negative Exponents	57	9.1 Solving Absolute Value Equations	155
3.4 Roots	59	9.2 Solving Absolute Value from a Number Line	158
3.5 Fractional Exponents	60	9.3 Solving and Graphing Absolute Value Inequalities	160
Section 3 Review	61	9.4 Interpreting Absolute Value Graphs	162
<b>Section 4</b>		9.5 Solving Absolute Value Word Problems	164
<b>The Language of Algebra</b>		Section 9 Review	165
4.1 Key Words in Word Problems	63	<b>Section 10</b>	
4.2 Rate Problems	67	<b>Polynomials</b>	
4.3 Using Rates in Equations	70	10.1 Introducing Polynomials	169
4.4 Setting up More Equations from Word Problems	75	10.2 Multiplying Monomials	170
4.5 Reverse Word Problems	78	10.3 Multiplying a Polynomial by a Monomial	172
4.6 Dimensional Analysis	80	10.4 Adding Polynomials	174
Section 4 Review	82	10.5 Subtracting Polynomials	176
<b>Section 5</b>		10.6 Multiplying Binomials	179
<b>Algebraic Equations</b>		Section 10 Review	183
5.1 Substitution Principle	85	<b>Section 11</b>	
5.2 The Addition Principle	87	<b>Rational Expressions</b>	
5.3 The Multiplication Principle	89	11.1 Dividing Monomials	185
Section 5 Review	94	11.2 Negative Exponents	187
<b>Section 6</b>		11.3 Rational Expressions to a Power	190
<b>Multi-Step Equations</b>		Section 11 Review	192
6.1 Introduction to Multi-Step Equations	95	<b>Section 12</b>	
6.2 Combining Like Terms	97	<b>Polynomial Applications</b>	
6.3 Variables on Both Sides	99	12.1 Perimeter and Circumference Word Problems	195
6.4 Equations with Parentheses	101	12.2 Area Word Problems	198
6.5 Equations with Decimals or Fractions	102	12.3 Area of Combined Shapes	201
6.6 Identifying Mistakes	105	12.4 Surface Area Word Problems	204
6.7 Solving Algebra Word Problems	109	12.5 Volume Word Problems	206
Section 6 Review	112	Section 12 Review	208

**Section 13****Factoring Polynomials**

13.1	Simple Factoring	211
13.2	Factoring Perfect Squares and Difference of Squares	214
13.3	Factoring Trinomials $x^2 + bx + c$	217
13.4	Factoring Trinomials $ax^2 + bx + c$	220
13.5	Prime Factors	224
13.6	Factoring Word Problems	226
Section 13 Review		229

**Section 14****Factoring Rational Expressions**

14.1	Factoring a Common Monomial	231
14.2	Factoring a Common Binomial	234
14.3	Rational Expression Word Problems	236
Section 14 Review		239

**Section 15****Quadratic Equations**

15.1	Solving Quadratic Equations by Factoring	241
15.2	Solving Quadratic Equations by Completing the Square	244
15.3	Solving Quadratic Equations by Using the Quadratic Formula	247
15.4	Using the Quadratic Discriminant	250
Section 15 Review		253

**Section 16****Quadratic Applications**

16.1	Choosing a Method to Solve Quadratics	255
16.2	Quadratic Word Problems: Geometric Shapes	257
16.3	Quadratic Word Problems: Motion Applications	260
16.4	Finding the Sum and Product of Quadratic Roots	263
Section 16 Review		265

**Section 17****The Coordinate Plane**

17.1	Introducing the Coordinate Plane	267
17.2	Plotting Points	268
17.3	Distance Formula	269
17.4	Midpoint Formula	272
17.5	Geometric Shapes on a Coordinate Plane	274
Section 17 Review		276

**Section 18****Linear Equations and Graphs**

18.1	Introducing Linear Equations	277
18.2	The Standard Form of a Line	278
18.3	Solving for a Point	284
18.4	Using a Table to Graph a Linear Equation	286
18.5	Graphing Horizontal and Vertical Lines	289
Section 18 Review		291

**Section 19****Linear Inequalities and Graphs**

19.1	Graphing Linear Inequalities	293
19.2	Reading Inequality Graphs	296
19.3	Linear Inequalities Applications	298
Section 19 Review		301

**Section 20****Slope**

20.1	Introducing Slope	303
20.2	Calculating Slope from Two Points	307
20.3	Slope-Intercept Form	309
20.4	Comparing Slope	312
20.5	Translating Lines	315
20.6	Finding an Equation Using Point and Slope	317
20.7	Finding an Equation Using Two Points	318
Section 20 Review		321

**Section 21****Slope as a Rate of Change**

21.1	Using Rise Over Run	325
21.2	Introducing Slope as a Rate of Change	327
21.3	Rate of Change in a Table	331
21.4	Using a Data Table to Find the Equation of a Line	333
21.5	Rate of Change on a Graph	336
21.6	Graphing Multiple Rates of Change	339
Section 21 Review		341

**Section 22****Problem Solving With Slope**

22.1	Recognizing the Graph from Given Data	343
22.2	Calculating an Equation from a Graph	345
22.3	Extrapolating by Using a Graph	347
22.4	Using Slope to Find Parallel Lines	350
22.5	Using Slope to Find Perpendicular Lines	354
22.6	Using Slope to Find Coincidental Lines	357
Section 22 Review		358

**Section 23****Matrices**

23.1	Introducing Matrices	361
23.2	Adding Matrices	362
23.3	Subtracting Matrices	365
23.4	Multiplying Matrices by a Scalar	368
Section 23 Review		372

**Section 24****Systems of Equations and Inequalities**

24.1	Introducing Systems of Equations	375
24.2	Solving Systems of Equations by Substitution	378
24.3	Solving Systems of Equations by Elimination	380
24.4	Systems of Inequalities in Slope-Intercept Form	384
24.5	Systems of Inequalities in Standard Form	387
24.6	Using Systems of Equations in Word Problems	389
Section 24 Review		392

**Section 25****Functions**

25.1	Introducing Functions	395
25.2	Types of Functions	398
25.3	Determining Domain and Range from a Table or Equation	401
25.4	Determining Domain and Range from a Graph	403
25.5	Linear Functions from Tables	408
Section 25 Review		410

<b>Section 26</b>		<b>Appendix, Scatter Plots with the Calculator</b>	
<b>Non-linear Functions</b>		(Calculator Instructions)	A-1
26.1 Interpreting Graphs and Tables of Quadratic Functions	413	<b>Index</b>	A-5
26.2 Quadratic Equations from Graphs and Tables	417	<b>Practice Test 1 (separate booklet)</b>	
26.3 Absolute Value Functions	421	Formula Sheet	PT1-5
26.4 Change in a Variable	425	Practice Test 1 Evaluation Chart	PT1-23
Section 26 Review	429		
<b>Section 27</b>		<b>Practice Test 2 (separate booklet)</b>	
<b>Scatter Plots</b>		Formula Sheet	PT2-5
27.1 Introducing Scatter Plots	433	Practice Test 1 Evaluation Chart	PT2-22
27.2 Determining Data Trends	435		
27.3 Making Predictions with Scatter Plots	440		
Section 27 Review	443		

# The Author

Jerald D. Duncan has been involved with education for the past thirty years. He has been a classroom teacher at the Middle School and High School levels, the assistant to the Vocational Director, Cobb County Schools, the Apprenticeship Coordinator, Cobb County Schools, and a curriculum materials author.

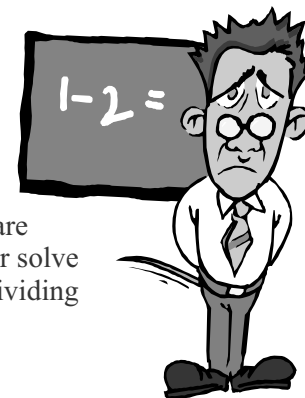
He is a graduate of Emmanuel College, Franklin Springs, Georgia, and Georgia State University in Atlanta.

Jerald is a nationally recognized Curriculum Development facilitator with curriculum projects completed in Alabama, Georgia, Michigan, and Pennsylvania. He has also conducted more than 40 teacher training workshops in over a dozen states in the areas of Applied Mathematics, Academic and Vocational Integration, Cooperative Learning, and Reading Across the Curriculum. He is also a CORD certified trainer in the areas of Applied Math, CORD Algebra, CORD Geometry, and Principles of Technology. Jerald is a frequent presenter at the SREB summer conferences and has presented at the Regional NCTM Conference.

Jerald has previously authored resource materials for Applied Math, CORD Algebra, CORD Geometry, Applied Biology/Chemistry, and Principles of Technology, and *Student Review Guide: Math*, Alabama High School Graduation Exam.

# Algebra Basics

## Section 1.3 Positive and Negative Numbers

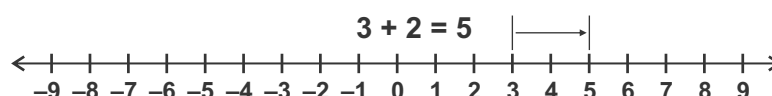


Algebra involves using variables to represent unknown values. Important skills in algebra are simplifying expressions and solving equations. In order to simplify algebraic expressions or solve algebraic equations, you need to know the rules for adding, subtracting, multiplying, and dividing positive and negative numbers.

### Adding Numbers

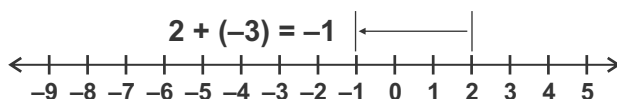
You've been adding numbers since first grade, but let's do a quick review on how number lines can be used. To add positive numbers, start at the first number. Then move from that position to the right by the number of units equal to the addend (or second number).

Look at this easy example. Start at 3 on the number line. The second number is 2, so move 2 places to the right. Wherever you end up is the sum. In the case of  $3 + 2$ , the sum is 5.



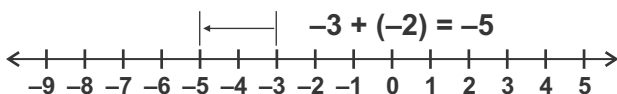
Negative numbers are to the left of zero on the number line. To add a negative number means to move left instead of right.

**Example 1:** What is the sum of  $2 + (-3)$ ?



Start by finding the first number, 2, on the number line. Now look at the second number,  $-3$ . The negative sign indicates that you move left instead of right. Move three units to the left. The result is  $-1$ .

**Example 2:** What is the sum of  $-3 + (-2)$ ?



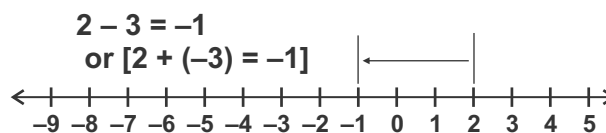
The first number is  $-3$ , so find it on the number line. It is to the left of zero since it is negative. Now add  $-2$ . Again, the negative sign indicates that you move left instead of right. The sum is  $-5$ .

### Subtracting Numbers

Subtraction means “take away,” and the subtraction symbol ( $-$ ) often is read as *take away* or *minus*. When you subtract two positive numbers, you take the second from the first. The subtraction symbol, just like a negative sign, means *move in the opposite direction* or *reverse*.

**Example 3:** What is the difference of  $2 - 3$ ?

Going back to the number line, the first number is positive, so start at positive 2. The second number is also positive, but the subtraction sign means “reverse.” Rather than moving to the right, you move three units to the left.



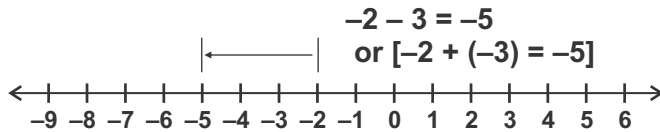
Compare this example to Example 1 above where a negative number was added. Can you see that subtracting a positive number is the same as adding a negative number?

## Section 1.3, continued

### Positive and Negative Numbers

Subtracting a positive number from a negative number is like adding two negative numbers.

**Example 4:** What is the difference of  $-2 - 3$ ?



Start at  $-2$ . The second number is positive, but the subtraction sign means “reverse.” Instead of moving to the right, you move three units to the left. The difference is  $-5$ .

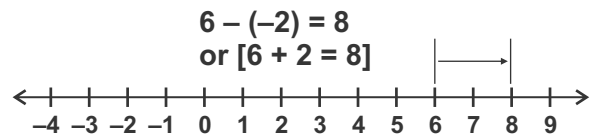
Now, the fun begins. Subtracting a negative number is the same as adding a positive number. Look at the examples below and use the number line to see why.

**Example 5:** What is  $6 - (-2)$ ?

Start at positive 6.

The second number is  $-2$ , which would normally mean you move to the left. However, the subtraction sign means you *reverse* the direction. When you reverse the move to the left, you move right instead.

The result is positive 8.

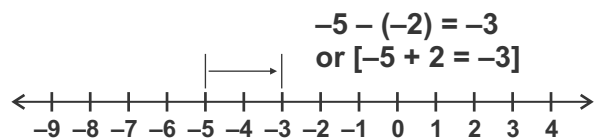


Subtracting two negative numbers is done the same way.

**Example 6:** What is  $-5 - (-2)$ ?

The first number is  $-5$ , so start there.

The second number is  $-2$ , but not so fast. The subtraction symbol means *reverse*, so go to the right instead. See how this problem is the same as  $(-5 + 2)$ ?



## Multiplying Numbers

When you multiply two numbers, you get a **product**. But when the factors have signs, you also have to take them into account in determining the sign of the product. Don’t panic. It’s not as difficult as you think. There are two simple rules that will help you keep it straight. (Remember that the raised dot is also a symbol for multiplication.)

### Rule for Multiplying Numbers with the Same Signs

If signs are the same (+, + or −, −), the product is positive.

$$2 \cdot 3 = 6$$

$$-2 \cdot -3 = 6$$

### Rule for Multiplying Numbers with Different Signs

If signs are different (+, − or −, +), the product is negative.

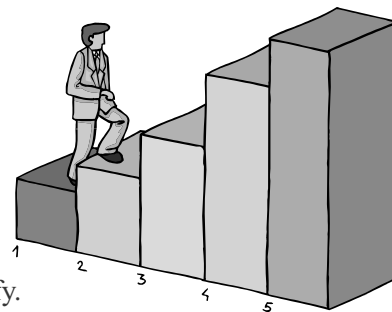
$$2 \cdot -3 = -6$$

$$-2 \cdot 3 = -6$$

# Multi-Step Equations

## Section 6.1

### Introduction to Multi-Step Equations



Only the most basic equations require the use of one principle (addition or multiplication) at a time. Most require the use of a combination of the two to simplify. That's what we mean by "multi-step." Equations that have constants on both sides of the equation and a coefficient in front of the variable take both the addition and multiplication principles. To solve, just take it one step at the time.

#### Example 1: Simplify the equation $3m + 4 = 13$ .

**Step 1:** First, use the addition principle to eliminate the +4 by adding the opposite.

$$\begin{array}{r} 3m + 4 = 13 \\ -4 = -4 \\ \hline 3m = 9 \end{array}$$

**Step 2:** Next, use the multiplication principle to eliminate the coefficient of the variable. Remember, multiply by the reciprocal.

$$\begin{array}{r} \frac{1}{3} \cdot 3m = 9 \cdot \frac{1}{3} \\ \frac{3}{3} m = \frac{9}{3} \\ m = 3 \end{array}$$

**Step 3:** Use the substitution principle to check the solution.

**Check:**

$$\begin{array}{r} 3(3) + 4 = 13 \\ 9 + 4 = 13 \\ 13 = 13 \end{array}$$

Now look at an example that doesn't work out so neatly.

#### Example 2: Simplify the equation $\frac{2}{3}x - 1 = 4$ .

**Step 1:** In this example, first eliminate the -1 by adding +1 to both sides.

$$\begin{array}{r} \frac{2}{3}x - 1 = 4 \\ +1 = +1 \\ \hline \frac{2}{3}x = 5 \end{array}$$

**Step 2:** Next eliminate the coefficient of  $x$  by multiplying by its reciprocal. The answer is an improper fraction, so convert to a mixed number as the final solution.

$$\begin{array}{r} (\frac{3}{2})(\frac{2}{3})x = (\frac{5}{1})(\frac{3}{2}) \\ x = \frac{15}{2} \text{ or } 7\frac{1}{2} \end{array}$$

**Step 3:** Use the improper fraction form to check the solution.

**Check:**

$$\begin{array}{r} (\frac{2}{3})(\frac{15}{2}) - 1 = 4 \\ 5 - 1 = 4 \\ 4 = 4 \end{array}$$

Section 6.1, continued  
Introduction to Multi-Step Equations

Practice

Solve the following multi-step equations by using the addition principle and the multiplication principle. Show your work and write your final answer in the blank provided. Convert improper fractions to mixed numbers. Be sure you check each solution by using substitution.

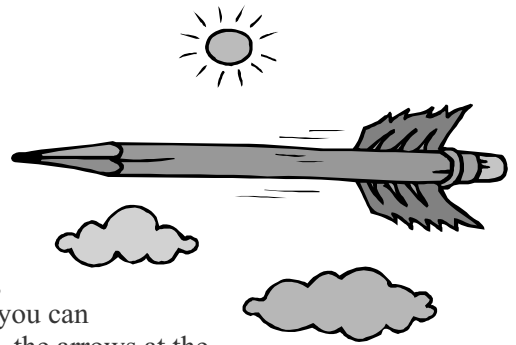
1. $3m - 2 = 7$ _____	2. $2a + 5 = -2$ _____	3. $\frac{1}{3}x + 2 = 2$ _____
4. $14c + 2 = 4$ _____	5. $-2x - 4 = 6$ _____	6. $-\frac{1}{2}m - 3 = 2$ _____
7. $-2x + 1 = -6$ _____	8. $3y - 7 = 1$ _____	9. $-\frac{2}{3}a + 5 = 1$ _____
10. $-\frac{3}{4}y - 1 = 5$ _____	11. $\frac{2}{5}x + 2 = -1$ _____	12. $\frac{2}{3}c - 9 = -2$ _____



# Inequalities

## Section 8.3

### Graphing Inequalities



To show the solution set for an inequality, you can also draw a graph. When the solution set has only one variable, the graph is a number line. A number line is a single line that runs infinitely in both directions — positive numbers, negative numbers, and zero included. You don't always see all three because you can draw only the portion of the number line that is needed for your solution. But, the arrows at the ends of the number line indicate it keeps going in both directions to include all numbers.

The solution on a number line begins with a circle as the starting point. To indicate “greater than” or “less than,” you use an open circle. To show “greater than or equal to” or “less than or equal to,” you use a solid circle. To indicate the direction of numbers that satisfy the inequality, a thicker line is drawn on or above the number line. Look at the examples below.

Open Circle $>, <$	Solid Circle $\geq, \leq$
$a > 5$	$a \geq 5$
$a < 5$	$a \leq 5$

#### REMEMBER!

If the comparison is “greater than” or “less than,” leave the circle open.  
If the comparison is “greater than or equal to” or “less than or equal to,” color in the circle.

**Hint:** Do you notice anything about the direction of the solution set in comparison with the inequality sign? As long as the variable is on the left, the inequality sign will point in the direction of the solution set. Caution: This only works if the inequality is completely solved and the variable is on the left.

#### Example 1: Graph the solution for the inequality $-2(x + 2) < 6$ .

Before you can graph, you must solve the inequality.

$$-2(x + 2) < 6$$

**Step 1:** Use the distributive property to clear the parentheses.

$$-2x - 4 < 6$$

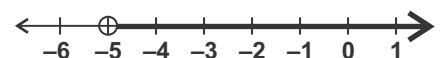
**Step 2:** Eliminate the  $-4$  by using the addition principle.

$$-2x < 10$$

**Step 3:** Eliminate the  $-2$  by using the multiplication principle. Reverse the sign when multiplying by a negative.

$$x > -5$$

**Step 4:** Now you can graph the solution. Draw an open circle around  $-5$ . The thick line points to the right, the same direction as the inequality symbol.



## Section 8.3, continued

### Graphing Inequalities

**Example 2:** Graph the solution for the inequality  $-1 \leq -x + 2$ .

$$-1 \leq -x + 2$$

**Step 1:** Use the addition principle to eliminate the +2.

$$-3 \leq -x$$

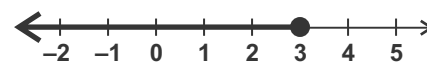
**Step 2:** Eliminate the minus sign by using the multiplication principle. Reverse the sign when multiplying by a negative.

$$3 \geq x$$

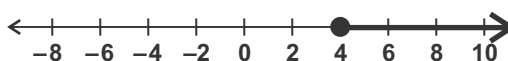
**Step 3:** Rearrange the inequality so that the variable is on the left.

$$x \leq 3$$

**Step 4:** Now you can graph the solution. Draw a solid circle around 3. The thick line points to the left, the same direction as the inequality symbol.



**Example 3:** The solution for an inequality is graphed on the number line below.



Which solutions of the following inequalities are represented by the graph?

$$2x - 2 \geq 6$$

$$8 \geq 2x$$

$$-x + 2 \leq 6$$

$$-x + 4 \leq 0$$

This problem is similar except you have to work backwards.

First, look at the graph. What is the solution represented by the graph? Hopefully you recognize that it is  $x \geq 4$ .

$$x \geq 4$$

Now simplify each inequality to see which ones match. The steps for simplifying each are shown below.

$$\begin{array}{r} 2x - 2 \geq 6 \\ +2 \quad +2 \\ \hline 2x \geq 8 \\ \frac{2x}{2} \geq \frac{8}{2} \end{array}$$

$$x \geq 4$$

yes

$$\begin{array}{r} 8 \geq 2x \\ \frac{8}{2} \geq \frac{2x}{2} \\ 4 \geq x \end{array}$$

$$x \leq 4$$

no

$$\begin{array}{r} -x + 2 \leq 6 \\ -2 \quad -2 \\ \hline -x \leq 4 \\ \times(-1) \quad \times(-1) \end{array}$$

$$x \geq -4$$

no

$$\begin{array}{r} -x + 4 \leq 0 \\ -4 \quad -4 \\ \hline -x \leq -4 \\ \times(-1) \quad \times(-1) \end{array}$$

$$x \geq 4$$

yes

From this example, you can see how easy it would be to make a small mistake to give you a wrong answer. Be careful when simplifying inequalities. Write down each step.

# Rational Expressions

## Section 11.2 Negative Exponents



So far all the division of monomials you have seen has been with positive exponents. What happens when the rational expression has negative exponents? You pray. Just kidding. Negative exponents aren't really that bad. You just have to pay attention — very careful attention. Let's start with a short review of what you already know about negative exponents.

### Rules for Negative Exponents

$$y^{-7} \rightarrow \frac{1}{y^7}$$

You've already seen that you can make a negative exponent positive by moving the variable to the denominator. But what if you already have a rational expression and the negative exponents are in the numerator or denominator? You move the variables. Here's how.

$$\frac{x^{-3}y^2}{y^2} \rightarrow \frac{y^2}{x^3y^2}$$

If the negative exponent is in the numerator, you *move* the variable to the denominator. If the negative exponent is in the denominator, move the variable to the numerator. It's just that simple. When you move variables with negative exponents, the exponents become positive.

$$\frac{x^2}{x^3y^{-2}} \rightarrow \frac{x^2y^2}{x^3}$$

$$\frac{2x^{-2}y^{-2}}{3x^{-3}y^{-4}} \rightarrow \frac{2x^3y^4}{3x^2y^2}$$

If you have all negative exponents in the numerator and the denominator, the variables swap places. Be sure you don't swap the coefficients; they already have a positive exponent. They're raised to the power of +1.

### Negative Exponents in Rational Expressions

If a rational expression has variables with negative exponents, use the rules above to make them positive. Once you make the exponents positive, you can simplify the rational expression by canceling common factors. Take a look at these examples.

**Example 1:** Simplify the expression  $\frac{16x^{-3}y^4z^{-2}}{12x^2y^2z^3}$ .

Since there are variables with negative exponents in the numerator, you *move* those variables to the denominator. Once all the exponents are positive, you can add the exponents that have the same base.

$$\frac{16x^{-3}y^4z^{-2}}{12x^2y^2z^3}$$

**Step 1:** Move the variables with negative exponents to the denominator and make the exponents positive.

$$\frac{16y^4}{12x^2x^3y^2z^3z^2}$$

**Step 2:** Add the exponents with the same bases.

$$\frac{16y^4}{12x^{2+3}y^2z^{3+2}} \rightarrow \frac{16y^4}{12x^5y^2z^5}$$

**Step 3:** Factor the coefficients if you can.

$$\frac{4 \cdot 4y^4}{4 \cdot 3x^5y^2z^5}$$

**Step 4:** Cancel the common factors in the coefficients and use the shortcut to cancel exponents.

$$\frac{4y^2}{3x^5z^5}$$

**Step 5:** After canceling, regroup what's left.

## Section 11.2, continued

### Negative Exponents

If you have variables with negative exponents in the denominator, you use the same process. Move the variables to the numerator to make the exponents positive, and then add the exponents with the same bases.

**Example 2:** Simplify the expression  $\frac{2xy^4z}{4x^2y^{-3}z^{-2}}$ .

**Step 1:** Move the variables with negative exponents from the denominator to the numerator and make the exponents positive.

**Step 2:** Add the exponents with the same bases. The tricky part here is adding the exponents for the  $z$ 's. Don't forget that a variable by itself has a power of 1. When simplifying the  $z$ 's, remember to add 1 to the other exponent of 2.

**Step 3:** Factor the coefficients.

**Step 4:** Cancel the common factors in the coefficients and use the shortcut to cancel exponents.

**Step 5:** After canceling, regroup what's left.

$$\begin{aligned} & \frac{2xy^4z}{4x^2y^{-3}z^{-2}} \\ & \frac{2xy^4y^3zz^2}{4x^2} \\ & \frac{2xy^{4+3}z^{1+2}}{4x^2} \longrightarrow \frac{2xy^7z^3}{4x^2} \\ & \frac{2xy^7z^3}{2 \cdot 2x^2} \\ & \frac{y^7z^3}{2x} \end{aligned}$$

Ready for a new wrinkle? Let's put variables with negative exponents in the numerator and the denominator. How's that for really ugly — or is it?

**Example 3:** Simplify the expression  $\frac{6x^{-3}y^4z^2}{9x^{-2}y^{-2}z^3}$ .

In this one, you have variables with negative exponents in both the numerator and the denominator. Use the same rules as before.

**Step 1:** Move the  $y$  in the denominator to the numerator and make the exponent positive.

**Step 2:** Swap the  $x$ 's in both the numerator and the denominator to make the exponents positive. Simplify the numerator.

**Step 3:** Factor the coefficients.

**Step 4:** Cancel the common factors in the coefficients and use the shortcut to cancel exponents.

**Step 5:** After canceling, regroup what's left.

$$\begin{aligned} & \frac{6x^{-3}y^4z^2}{9x^{-2}y^{-2}z^3} \\ & \frac{6x^2y^4y^2z^2}{9x^3z^3} \\ & \frac{6x^2y^{4+2}z^2}{9x^3z^3} \longrightarrow \frac{6x^2y^6z^2}{9x^3z^3} \\ & \frac{2 \cdot 2x^2y^6z^2}{3 \cdot 3x^3z^3} \\ & \frac{2y^6}{3xz} \end{aligned}$$

# Slope as a Rate of Change

## Section 21.2 Introducing Slope as a Rate of Change

A rate of change compares the change in a dependent variable to the change in an independent variable. Think of it as a fraction with units. The word “per” is often used to make the comparison. Rates represent slopes.

miles **per** gallon

$\frac{\text{miles}}{\text{gallon}}$

cost **per** item

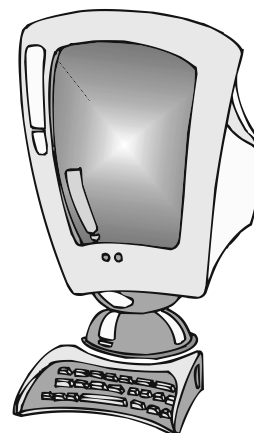
$\frac{\text{cost}}{\text{item}}$

feet **per** second

$\frac{\text{feet}}{\text{second}}$

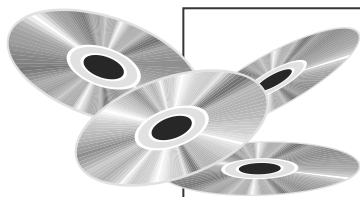
parts **per** million

$\frac{\text{parts}}{\text{million}}$



Rate is the dependent quantity divided by the independent quantity. When you graph the relationship, the dependent quantity becomes the  $y$ -value, and the independent quantity becomes the  $x$ -value. What’s the difference between a dependent variable and an independent variable? Consider the following:

$$\frac{\Delta y}{\Delta x} = \frac{\text{Dependent Quantity}}{\text{Independent Quantity}}$$



“I want to buy some cd’s,” said Serina. “How much are they?”  
 “Nine dollars each,” replies the clerk.  
 “How much will that be?” Serina asks.  
 “Depends,” responds the clerk, “on how many you buy.”  
 “So cost depends on how many cd’s?” Serina asks.  
 “Exactly,” the clerk says.

What does all that mean? The change in the dependent quantity “depends” on the change in the independent quantity. In the example, total cost *depends* on the number of cd’s, so cost is the dependent variable, and the number of cd’s is the independent variable. So what does that have to do with slope? Slope equals the rate of change.

### Practice 1

Determine the independent and dependent variables in each of the following. Label the independent variable with an  $I$  and the dependent variable  $D$ .

**Example:** A sports utility vehicle gets about 18 miles per gallon.

  D   miles        I   gallons

1. The speed of light is given in miles per second.

       miles             second

2. The post office charges by the ounce to mail a letter.

       ounce             cost

3. The weight of an infant is monitored over several months.

       weight             months

4. Every five minutes, a machine produces three widgets.

       time             widgets

5. At summer camp, every 4 student campers must have 1 adult camp leader.

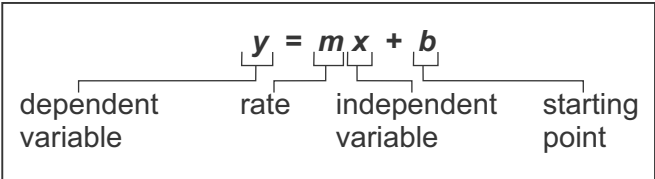
       campers             leaders

Section 21.2, continued  
Introducing Slope as a Rate of Change

Now that you understand the difference between dependent and independent variables, let’s take a look at what linear equations can represent in real-world problems.

In Section 4.3, you worked with algebraic word problems that use rates. The equation that represents the rate problem is a linear equation. Each part of the equation relates to the graph of the resulting line.

The  $y$  is the dependent variable, the slope is the rate, the  $x$  is the independent variable, and the  $b$  is the starting point when the independent variable is zero (which is the  $y$ -intercept).



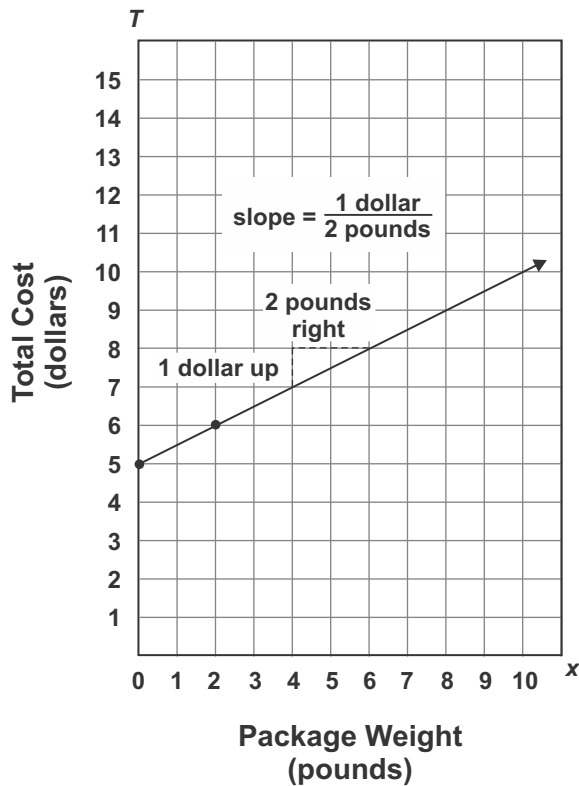
**Example 1:** A company charges a flat fee of \$5.00 to ship a package plus \$0.50 per pound the package weighs. Write an equation that represents the total cost,  $T$ , for shipping a package weighing  $x$  pounds. Graph the line.

You saw the equation for this problem in Section 4.3, but now, let’s review what each part of the equation stands for in terms of a linear equation.

$$T = 5 + 0.5x$$

First, let’s rewrite the equation in slope-intercept form. To make the problem easier to see, rewrite the decimal as a fraction.

$$T = \frac{1}{2}x + 5$$



When you graph the line, you can see the relationships of each term in the equation.

$T$ , the total cost, is the dependent variable. Total cost depends on package weight,  $x$ . The  $T$  could be designated  $y$  instead, but in real-world problems, the dependent variable is not always  $y$ , and the independent variable is not always  $x$ . The dependent variable is plotted along the vertical axis of the graph.

In this case,  $x$  is the independent variable, and it is plotted on the  $x$ -axis of the graph.

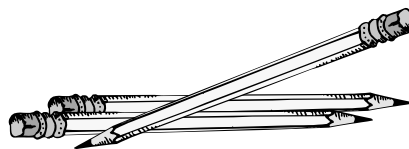
The rate is \$0.50 per pound, or in fraction form, it is 1 dollar per 2 pounds. That means the slope of the line is 1 over 2.

The starting point is \$5.00. The starting point tells us that even if the package weighed zero pounds, the flat fee of \$5.00 would still be the cost. That means the minimum cost is \$5.00. \$5.00 is the  $T$ -intercept.

# Functions

## Section 25.2

### Types Of Functions



To this point, all the graphs you have seen have been linear. Linear equations are functions as long as they are not vertical lines. But as you saw in the last sub-section, linear is not the only type of equation that can be a function. The other types you will most commonly encounter are quadratic, absolute value, and exponential. There's even one called a step function, but we'll get to that.

### Vertical Line Test

To the right is a graph of a linear equation. It's a function because if you picked a set of points from the line, no value of  $x$  would have more than one value of  $y$ .

Another way you can tell, just from a graph, is called the **vertical line test**. If you draw a vertical line anywhere on the grid and it intersects the graph in only one place, the graph is a function.

Another way to think of the vertical line test is the "pencil test." Lay your pencil across the graph vertically. If it's a function, it will intersect the graph at only one point no matter where you move it across the graphed line.

Any vertical line you draw through the graph of a linear equation will cross at only one point. Let's check some other types of graphs.

### Quadratic Equations

When you graph a quadratic equation, the graph is a **parabola**. This particular quadratic equation happens to be  $y = x^2$ . It opens upward because the coefficient of  $x$  is positive.

If you use the vertical line test, quadratic equations in the form of  $y = ax^2 + bx + c$ , when  $a \neq 0$ , are functions. Any vertical line on the graph would pass through only one point.

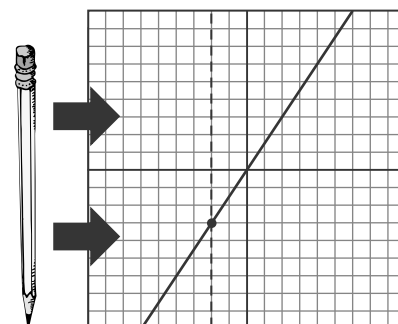
When a quadratic equation has the  $y$  squared instead of the  $x$ , you also get a parabola. However, this time the parabola is not a function. Consider the equation  $y^2 = x$ . If you solve for  $y$  by taking the square root of both sides, you get  $y = \pm\sqrt{x}$ . Remember, when you take a square root, you have to consider both the positive and negative values. The graph of  $y = +\sqrt{x}$  would be the top half of the parabola. To get the bottom half, you also have to graph  $y = -\sqrt{x}$ .

Apply the vertical line test to this graph and see what you get. The line crosses the graph in more than one place. That means it's not a function.

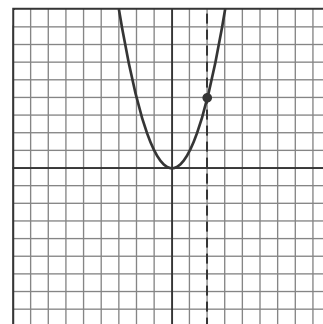
### Absolute Value Equations

Similar to taking the square root of an equation, you also have to consider both the positive and negative cases when you evaluate absolute value. But this time, the graph is a bit different. The graph to the right shows  $y = |x|$ .

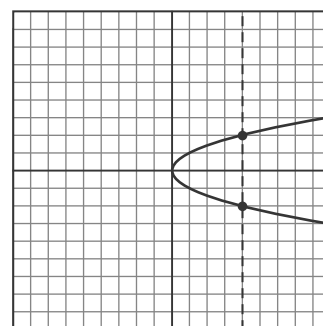
As you can see, the vertical line test confirms that this absolute value equation is a function. All absolute value equations are functions as long as they are in the form  $y = a|x + b| + c$ . ( $x = |y|$  would not be a function.)



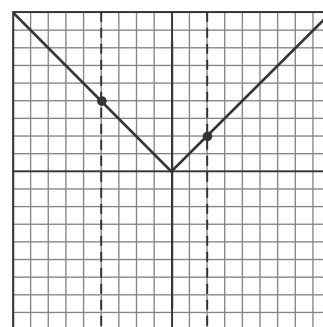
Linear



Quadratic ( $x^2$ )



Quadratic ( $y^2$ )



Absolute Value

Section 25.2, continued  
Types Of Functions

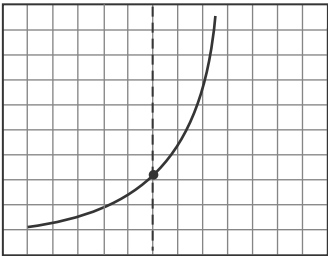
Exponential Equations

Exponential equations, when graphed, produce a curve similar to this one. Two of the most common forms of exponential equations are *exponential growth* and *exponential decay*. In exponential equations, there is a product of constants with one of them raised to a power of the independent variable.

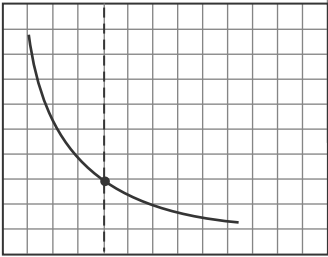
$$y = a \cdot b^x$$

where  $a$  is a nonzero constant,  
 $b > 0$  and  $b \neq 1$ , and  $x$  is a real number.

Sounds complicated, doesn't it? Fortunately, all you need to do is determine if the graph of an exponential equation is a function by using the vertical line test. When you graph exponential growth, the curve slopes upward. An exponential decay curve slopes down. And, as you can see, both are functions.



Exponential Growth

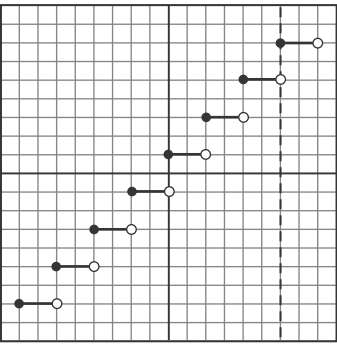


Exponential Decay

Step Functions

You may not hear much about step functions in Algebra 1, but you may be asked to determine if the graph is a function. The graph of a step function is nothing but horizontal line segments that are disconnected from one another. The key is that the *segments do not overlap*. As you may recall from number line graphs, the open circle on the end of the segment means that point is not included in the graph. So, as you can see, any vertical line would intersect the graph at only one point.

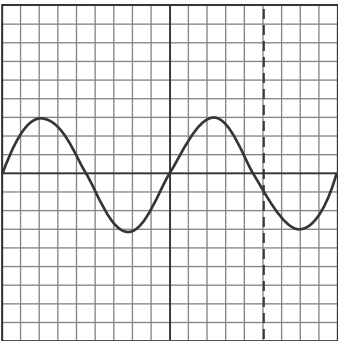
If you encounter a graph like this, just be aware that it will pass the vertical line test *iff* (if and only if) the line segments do not overlap. That makes it a function.



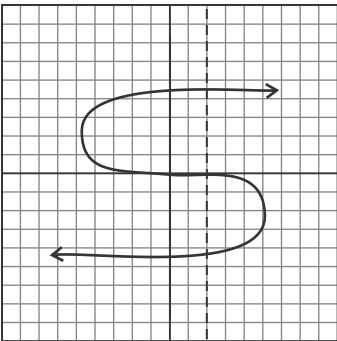
Unusual Functions

Even if you are given graphs that you don't recognize, don't panic. The vertical line test will determine whether the graph is a function.

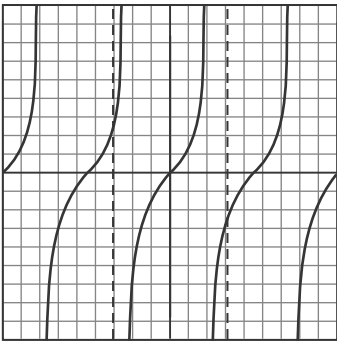
Function



Not a Function



Function





# Algebra 1

## DIRECTIONS

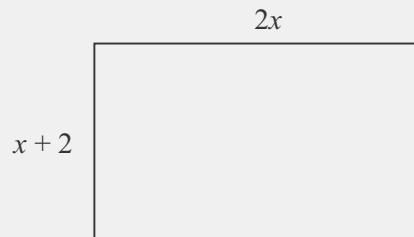
Read each problem carefully. Then work the problem and find your answer among the answer choices.

### SAMPLE A

What value of  $x$  makes the equation  $x + 4 = 9$  true?

- A** 5
- B** 6
- C** 9
- D** 13

### SAMPLE B



Which of these is equivalent to the perimeter of this rectangle?

- A**  $3x + 2$
- B**  $6x + 4$
- C**  $2x^2 + 4x$
- D**  $2x^2 + 2$

1. Which of the following is equivalent to the expression shown below?

$$\frac{8x^{-2}y^2z^{-2}}{6x^{-4}y^{-8}z^{-3}}$$

- A  $\frac{3}{4x^2y^{10}z}$
- B  $\frac{3x^2y^{10}}{4z}$
- C  $\frac{4x^2y^6}{3z}$
- D  $\frac{4x^2y^{10}z}{3}$

(A) (B) (C) (D)

4. Which of the following is equivalent to the expression below?

$$\frac{2ab^2}{6a^2b^2 + 2ab^2 + 4a^2b}$$

- A  $\frac{1}{6a^2b^2 + 4a^2b}$
- B  $\frac{1}{3a + 1 + 2ab}$
- C  $\frac{b}{3ab + b + 2a}$
- D  $\frac{b}{5a}$

(A) (B) (C) (D)

2. A small mail order catalog company pays a flat rate of \$175 per year for a bulk mail permit to send out its catalogs plus a charge of \$1.06 per catalog. If the company has an advertising budget of \$2500, what is the maximum number of catalogs they can send out in a year without going over their budget?

- A 2193
- B 2194
- C 2195
- D 2325

(A) (B) (C) (D)

5. What is the product of the solutions to the following quadratic equation?

$$2x^2 - 5x - 12 = 0$$

- A -12
- B -6
- C 0
- D 5

(A) (B) (C) (D)

3. What is the equation of the line that contains the points  $(-2, 0)$  and  $(3, 5)$ ?

- A  $y = x - 2$
- B  $y = x + 2$
- C  $y = 5x + 2$
- D  $y = 5x - 2$

(A) (B) (C) (D)

6. Which of the following is  $6y^2 - 8y + 10$  factored completely over the set of rational numbers?

- A  $2(3y - 5)(y - 1)$
- B  $2(3y - 1)(y + 5)$
- C  $2(3y + 1)(y - 5)$
- D  $2(3y^2 - 4y + 5)$

(A) (B) (C) (D)

35. Which of the following is equivalent to the expression shown below?

$$(2x + 3y - 4) - 5(3x - y - 2)?$$

- A  $-13x + 8y + 6$
- B  $-13x + 8y - 14$
- C  $-6x - 2y + 6$
- D  $-6x - 2y - 14$

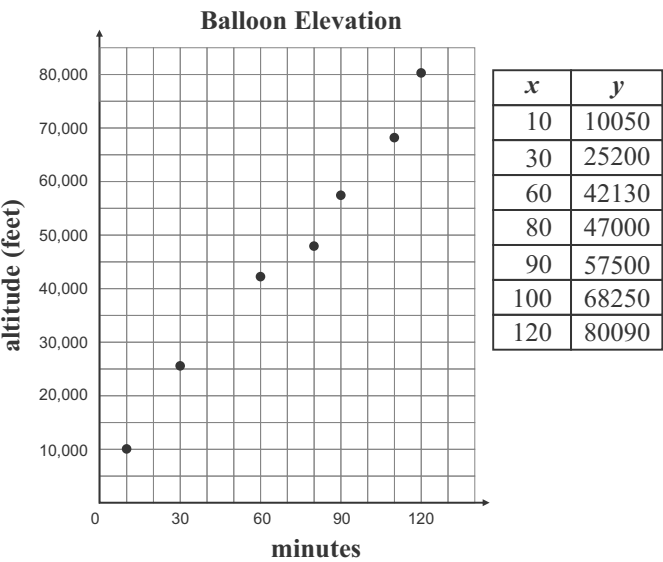
(A) (B) (C) (D)

37. A certain function is represented by  $f(x) = -2x + 4$ . If the range of this function is  $\{0, 2, 4\}$ , what is the domain of the function?

- A  $\{-4, 0, 4\}$
- B  $\{6, 8, 12\}$
- C  $\{-2, 2, 4\}$
- D  $\{0, 1, 2\}$

(A) (B) (C) (D)

36. The scatter plot below show the altitude of a weather balloon as it ascends into the atmosphere for two hours.

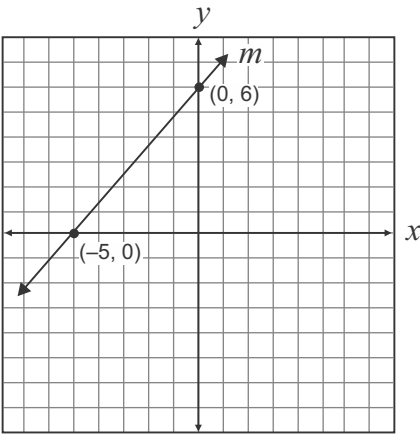


Which equation is closest to the line-of-best fit for this data?

- A  $y = 500x + 10$
- B  $y = 5000x + 742$
- C  $y = 759x + 5130$
- D  $y = 613x + 4264$

(A) (B) (C) (D)

38. Line  $m$  is shown on the graph below.



Which equation describes a line that is *parallel* to line  $m$ ?

- A  $y = -\frac{6}{5}x - 6$
- B  $y = \frac{5}{6}x + 6$
- C  $y = \frac{6}{5}x + 6$
- D  $y = -\frac{6}{5}x + 6$

(A) (B) (C) (D)

# Algebra 1

## Practice Test 1

### Evaluation Chart

If you missed question #:	Go to section(s):	If you missed question #:	Go to section(s):
1	3.1, 3.3, 11.1, 11.2	36	27.2
2	4.3, 8.4	37	25.3
3	17.1, 20.1, 20.2, 20.3, 20.6, 20.7	38	20.3, 22.4
4	13.1, 14.1	39	20.3, 22.5
5	13.4, 15.1, 16.4	40	27.2
6	13.1, 13.5, 15.4	41	23.2
7	7.2, 10.4, 10.6, 12.2	42	4.1, 24.2, 24.3, 24.6
8	20.3, 20.4	43	2.1, 10.3, 10.4, 10.5
9	5.2, 5.3, 6.1, 9.1	44	7.2, 7.4
10	19.1, 24.4, 24.5	45	7.2, 10.6, 26.4
11	20.2, 20.3, 20.7, 21.3, 21.4	46	15.2
12	4.1, 7.5	47	17.3
13	9.1, 9.3	48	25.1
14	27.2	49	17.4, 17.5
15	8.1, 8.2, 8.3	50	10.4, 12.1
16	21.2	51	27.2
17	13.4, 13.5	52	8.4, 8.5
18	20.3, 20.6	53	13.4
19	13.2, 13.6	54	27.1
20	10.4	55	8.1, 8.2
21	26.1, 26.2	56	23.2, 23.3
22	17.2, 20.1	57	1.4
23	13.4, 15.1	58	25.1, 25.4
24	21.2	59	27.2
25	2.1, 10.3, 10.4, 10.5	60	21.2, 21.6
26	4.3, 6.7	61	7.1, 7.5
27	23.4	62	13.4, 14.2, 14.3
28	20.3, 20.4	63	2.1
29	19.1, 19.2	64	1.3, 1.4, 2.2, 2.3
30	22.4, 22.5, 22.6, 24.1	65	21.2, 21.5, 22.1, 22.2
31	21.2, 21.6		
32	26.1		
33	26.3		
34	27.3		
35	2.1, 10.3, 10.4, 10.5		